Review and Preview to Chapter 3

1. $||3x+1|-x| = 2$

$|3x+1| = x+2$ or $|3x+1| = x-2$

$3x+1 = 2+x$ or $3x+1 = -2-x$ or $3x+1 = x-2$ or $3x+1 = 2-x$

$2x = 1$ or $4x = -3$ or $2x = -3$ or $4x = 1$

$x = \frac{1}{2}$ or $x = -\frac{3}{4}$ or $x = -\frac{3}{2}$ or $x = \frac{1}{4}$

$x = -\frac{3}{2}$ and $x = \frac{1}{4}$ do not satisfy the equation, so the solution is $x = \frac{1}{2}$ or $x = -\frac{3}{4}$

2. Using a calculator, $\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}} \doteq 2$. To solve, we will try to find $\left(\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}\right)^2$,

$$\left(\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}\right)^2 = \frac{2+2\sqrt{12}+6}{2+\sqrt{3}} = \frac{8+4\sqrt{3}}{2+\sqrt{3}} = \frac{4(2+\sqrt{3})}{2+\sqrt{3}} = 4.$$

So $\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}$ must be $\sqrt{4} = 2$.

3. $v_1 = 80$ km/h, $v_2 = 50$ km/h. Let t_1 be the time taken to make the trip at speed v_1 and let t_2 be the time taken to make the trip at speed v_2.

$d = v_1t_1 = v_2t_2 \Rightarrow 80t_1 = 50t_2 \Rightarrow t_2 = \frac{8}{5}t_1$

$$\text{speed} = \frac{\text{total distance}}{\text{total time}} = \frac{v_1t_1 + v_2t_2}{t_1+t_2} = \frac{v_1t_1+v_1t_1}{t_1+\frac{8}{5}t_1} = \frac{80t_1+80t_1}{\frac{13}{5}t_1} = \frac{160t_1}{\frac{13}{5}t_1} = \frac{800}{13}$$

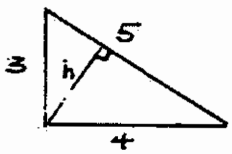
$\doteq 61.5$ km/h. So the average speed for the round trip is 61.5 km/h.

4. Let h be the length of the altitude. The other side has length $\sqrt{5^2-3^2} = 4$. The area of the triangle is $\frac{1}{2}\cdot 4\cdot 3 = 6$. But the area of the triangle is also $\frac{1}{2}\cdot 5\cdot h = 6$, so $h = 2.4$ and hence the length of the altitude is 2.4 cm.

[*Alternate solution*:

Use similar triangles:

$\frac{3}{5} = \frac{h}{4} \Rightarrow h = \frac{12}{5} = 2.4$]



5. The odometer reading is proportional to the number of tire revolutions. Let r_1 represent the number of revolutions made by the tire on the 640 km trip and r_2 represent the number of revolutions made by the tire on the 625 km trip. Then $r_1 = 640k$ and $r_2 = 625k$ for some constant k. Let d be the actual distance travelled, R_1 be the radius of normal tires, and R_2 be the radius of snow tires. Therefore d

$= 2\pi R_1 r_1 = 2\pi R_2 r_2 \Leftrightarrow R_1 r_1 = R_2 r_2 \Leftrightarrow R_2 = \frac{R_1 r_1}{r_2} = \frac{(33)(640k)}{625k} = \frac{4224}{125} \doteq 33.8.$
So the radius of the snow tires is approximately 33.8 cm.

6. Let t_1 represent the time taken to fill the pool with Bob's hose alone and t_2 the time taken to fill the pool with Jim's hose alone. Together it takes them 18 hours to fill the pool, so in one hour they could fill 1/18 of the pool of which Bob would fill $1/t_1$ and Jim would fill $1/t_2$. Therefore $1/t_1 + 1/t_2 = 1/18 \Leftrightarrow$ $18(t_1 + t_2) = t_1 t_2$. We also know that Bob alone can fill the pool in 6 hours less than Jim alone, so $t_1 = t_2 - 6$ and substituting this gives

$18(t_2 - 6 + t_2) = (t_2 - 6)t_2 \Leftrightarrow t_2^2 - 42t_2 + 108 = 0$

$\Leftrightarrow t = \frac{42 \pm \sqrt{42^2 - 4 \cdot 1 \cdot 108}}{2 \cdot 1} = 21 \pm \frac{1}{2}\sqrt{1332} = 21 \pm \sqrt{333} = 21 \pm 3\sqrt{37}$

Since $21 - 3\sqrt{37} < 1$[illegible] it cannot be a solution and so $t_2 = 21 + 3\sqrt{37} \approx 39\frac{1}{4}$ and then $t_1 = t_2 - 6 = 15 + 3\sqrt{37} \approx 33\frac{1}{4}$. Thus it takes Bob $33\frac{1}{4}$ hours alone and Jim $39\frac{1}{4}$ hours alone.

Exercise 3.1

1. From the graph

(a) The initial velocity was 0. (b) The car was going faster at C.

(c) The car was speeding up at A and C, slowing down at B.

(d) The car is stopped.

(e) The car returned to the point at which it started.

2. (a) $s = 5 + 12t$; $v(t) = s'(t) = 12$, $v(2) = v(4) = 12$ m/s.

(b) $s = 8t^2 - 24t + 5$; $v(t) = 16t - 24$, $v(2) = 16(2) - 24 = 8$ m/s,
$v(4) = 16(4) - 24 = 40$ m/s.

(c) $s = t^3 - 6t^2$; $v(t) = 3t^2 - 12t$, $v(2) = 3(2)^2 - 12(2) = -12$ m/s,
$v(4) = 3(4)^2 - 12(4) = 0$ m/s.

(d) $s = \frac{5t}{1+t}$; $v(t) = \frac{5(1+t) - 5t}{(1+t)^2} = \frac{5}{(1+t)^2}$, $v(2) = \frac{5}{(1+2)^2} = \frac{5}{9}$ m/s,
$v(4) = \frac{5}{(1+4)^2} = \frac{1}{5}$ m/s.

3. $h = 80 - 15t - 4.9t^2$

$v(t) = -15 - 9.8t$, $v(1) = -15 - 9.8(1) = -24.8$ m/s, $v(2) = -15 - 9.8(2) = -34.6$ m/s

4. $h = 24.5t - 4.9t^2$

(a) $v(t) = 24.5 - 9.8t$, $v(1) = 24.5 - 9.8(1) = 14.7$ m/s, $v(2) = 24.5 - 9.8(2) = 4.9$ m/s,
$v(3) = 24.5 - 9.8(3) = -4.9$ m/s, $v(4) = 24.5 - 9.8(4) = -14.7$ m/s

(b) When the ball reaches its maximum height, velocity will be 0. $v(t) = 24.5 - 9.8t = 0$ when $t = 2.5$ s. So the ball reaches its maximum height after 2.5 s.

(c) Maximum height occurs when $t = 2.5$ s, so $h(2.5) = 24.5(2.5) - 4.9(2.5)^2 = 30.625$ m. So its maximum height is 30.625 m.

(d) The ball hits the ground when $h = 0$, so $24.5t - 4.9t^2 = 0 \Rightarrow t(24.5 - 4.9t) = 0 \Rightarrow t = 0$ or $t = 5$. So the ball hits the ground after 5 s.

(e) $v(5) = 24.5 - 9.8(5) = -24.5$ m/s. So the ball hits the ground at 24.5 m/s.

5. $s = 160t^2 + 20t$

$v(t) = s'(t) = 320t + 20 = 100 \Rightarrow 320t = 80 \Rightarrow t = 0.25$ h. So the velocity reaches 100 km/h at 15 min.

6. $s = t^3 - 3t^2 - 5t$

$v(t) = 3t^2 - 6t - 5 = 4 \Rightarrow 3t^2 - 6t - 9 = 0 \Rightarrow (t+1)(t-3) = 0 \Rightarrow t = -1$ s, $t = 3$ s.
Since t must be positive, the particle reaches 4 m/s at 3 s.

7. $s = t^2 - 4t + 4$

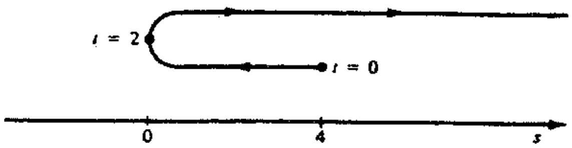
(a) $v(t) = 2t - 4$, $v(1) = 2 - 4 = -2$ m/s, $v(3) = 2(3) - 4 = 2$ m/s

(b) $v(t) = 0 = 2t - 4 \Rightarrow t = 2$ s. The particle is at rest at 2 s.

(c) $v(t) > 0 \Rightarrow 2t - 4 > 0 \Rightarrow t > 2$ s.

The particle moves in the positive direction after 2 s.

(d)

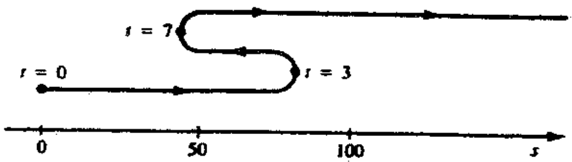


8. $s = t^3 - 15t^2 + 63t$

(a) $v(t) = 3t^2 - 30t + 63 = 0 \Rightarrow (t-3)(t-7) = 0 \Rightarrow t = 3$ s, $t = 7$ s. So the particle is at rest when $t = 3$ s, $t = 7$ s.

(b) $v(t) > 0 \Rightarrow (t-3)(t-7) > 0 \Rightarrow t - 3 > 0$ and $t - 7 > 0$ or $t - 3 < 0$ and $t - 7 < 0 \Rightarrow$ $t > 3$ and $t > 7$ or $t < 3$ and $t < 7 \Rightarrow t > 7$ s or $t < 3$s. So the particle is moving in the positive direction when $t > 7$ s or $t < 3$ s (but $t \geq 0$ s).

(c)



(d) Since the particle changes direction when $t = 3$ s and $t = 7$ s, the total distance must be divided up into three parts.

For $0 \leq t \leq 3$, distance $= s(3) - s(0) = 3^3 - 15(3)^2 + 63(3) - 0 = 81$ m.

For $3 \leq t \leq 7$, distance $= s(3) - s(7) = 81 - [7^3 - 15(7)^2 + 63(7)] = 81 - 49 = 32$ m.

For $7 \leq t \leq 10$, distance $= s(10) - s(7) = 10^3 - 15(10)^2 + 63(10) - 49 = 130 - 49 = 81$ m.

So the total distance travelled $= 81 + 32 + 81 = 194$ m.

9. $s = 450 + 10t - 5t^2$

(a) At its max. height, $v(t) = 10 - 10t = 0 \Rightarrow t = 1$ s. So the ball reaches its max. heigth after 1 s.

(b) $s(t) = 450 + 10t - 5t^2 = 0 \Rightarrow t = \dfrac{-10 \pm \sqrt{10^2 - 4(-5)(450)}}{2(-5)} = 1 \pm \sqrt{91}$

Since $t \geq 0$, $t = 1 + \sqrt{91} \doteq 10.5$ s. So the ball hits the ground after approx. 10.5 s.

(c) $v(t) = 10 - 10t$, $v(1 + \sqrt{91}) = 10 - 10(1 + \sqrt{91}) = -10\sqrt{91} \doteq -95.4$ m/s

So the ball hits the ground at approx. 95.4 m/s.

Exercise 3.2

1. From the graph

(a) O to A, positive (b) A to B, negative (c) B to C, positive

(d) C to D, zero (e) D to E, positive

2. From the graph

(a) The velocity is increasing from O to A since the slopes of the tangents are increasing. The acceleration is positive.

(b) (i) A to B, negative (ii) positive

(iii) C to D, zero (iv) D to E, negative

3. (a) $s = 12 + 30t$; $v = 30$, $a = 0$ (b) $s = 16t^2 + 5t - 10$; $v = 32t + 5$, $a = 32$

(c) $s = t^3 + 5t^2 + t + 1$; $v = 3t^2 + 10t + 1$, $a = 6t + 10$

(d) $s = \sqrt{t^2 + t}$; $v = \dfrac{2t+1}{2\sqrt{t^2+t}}$, $a = \dfrac{4\sqrt{t^2+t} - (2t+1)(t^2+t)^{-\frac{1}{2}}(2t+1)}{\left[2\sqrt{t^2+t}\right]^2}$

$= \dfrac{4(t^2+t) - (4t^2+4t+1)}{4\sqrt{(t^2+t)^3}} = \dfrac{-1}{4\sqrt{(t^2+t)^3}}$

4. (a) $s = 100 - 15t - 4.9t^2$; $v = -15 - 9.8t$, $a = -9.8$ m/s, $a(4) = -9.8$ m/s^2

(b) $s = t^3 - t^2$; $v = 3t^2 - 2t$, $a = 6t - 2$, $a(4) = 6(4) - 2 = 22$ m/s^2

(c) $s = t^3 - 2t^2 + 3t - 5$; $v = 3t^2 - 4t + 3$, $a = 6t - 4$, $a(4) = 6(4) - 4 = 20$ m/s^2

(d) $s = \dfrac{5t}{1+t}$; $v = \dfrac{5(1+t) - 5t}{(1+t)^2} = \dfrac{5}{(1+t)^2}$, $a = \dfrac{-10}{(1+t)^3}$,

$a(4) = \dfrac{-10}{(1+4)^3} = -0.08$ m/s^2

5. $s = s_0 + v_0 t + \frac{1}{2}gt^2$

(a) initial position, $s(0) = s_0 + v_0(0) + \frac{1}{2}g(0)^2 = s_0$

(b) initial velocity $v(0)$, $v(t) = v_0 + gt$, $v(0) = v_0$

(c) acceleration a, $a = g$

6. $s(t) = t^3 - 12t$; so $v(t) = 3t^2 - 12 = 0 \Rightarrow t = 2$ s ; $a(t) = 6t$, $a(2) = 12$ m/s^2. So the acceleration is 12 m/s^2 when the velocity is 0 m/s .

Exercise 3.2

7. $s = t^3 - 9t^2 + 18t$

(a) $v(t) = 3t^2 - 18t + 18$, $a(t) = 6t - 18 = 0$ when $t = 3$ s. So the acceleration is 0 m/s^2 when $t = 3$ s.

(b) $s(3) = 3^3 - 8(3)^2 + 18(3) = 0$ m ; $v(3) = 3(3)^2 - 18(3) + 18 = -9$ m/s

8. $s = t^4 - 12t^3 + 30t^2 + 5t$; so $v(t) = 4t^3 - 36t^2 + 60t + 5$, $a(t) = 12t^2 - 72t + 60$.
So $a(t) > 0 \Rightarrow 12t^2 - 72t + 60 > 0 \Rightarrow (t-1)(t-5) > 0 \Rightarrow t - 1 > 0$ and $t - 5 > 0$ or $t - 1 < 0$ and $t - 5 < 0 \Rightarrow t > 1$ and $t > 5$ or $t < 1$ and $t < 5 \Rightarrow t > 5$ s or $t < 1$ s ($t \geq 0$ s).
So the acceleration is positive when $t > 5$ s or $t < 1$ s ($t \geq 0$) and is negative when $1 < t < 5$ s.

9. $v_0 = 72$ km/h $= 72\ 000$ m/h $= 20$ m/s, $a = -12$ m/s^2

(a) Verify $v(t) = -12t + 20$; When $t = 0$ s, $v = 20$ m/s. The acceleration is -12 m/s^2. So $v(t) = -12t + 20$ gives the correct intial velocity and deceleration.

(b) Time to stop ; $v(t) = -12t + 20 = 0$ when $t = \frac{5}{3}$ s. So it takes $\frac{5}{3}$ s to stop.

Exercise 3.3

1. Rate of change of cube volume with respect to edge length x when $x=4$.
Volume of cube $V(x) = x^3$; therefore, $V'(x) = 3x^2$, so $V'(4) = 3(4)^2 = 48$ units^3/unit.

2. Rate of change of circle area with respect to radius r when $r=5$ cm.
Area of circle $A(r) = \pi r^2$; therefore, $A'(r) = 2\pi r$, so $A'(r) = 2\pi(5) = 10\pi$ cm^2/cm.

3. Rate at which water flows out of tank after 10 min.
$V(t) = 1000\left(1-\frac{t}{60}\right)^2$, $V'(t) = 2000\left(1-\frac{t}{60}\right)\left(-\frac{1}{60}\right) = -\frac{100}{3}\left(1-\frac{t}{60}\right)$
$V(10) = -\frac{100}{3}\left(1-\frac{10}{60}\right) = -\frac{250}{9}$ L/min. So the water is flowing out at $\frac{250}{9}$ L/min.

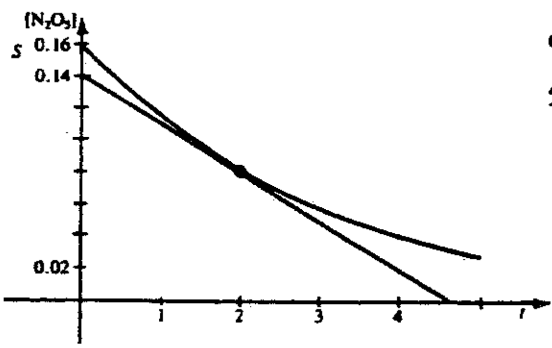
4. $m(x) = \sqrt{x}$ kg.
(a) Density $= \frac{\Delta m}{\Delta x} = \frac{m(1.1)-m(1)}{1.1-1} = \frac{\sqrt{1.1}-\sqrt{1}}{0.1} \doteq 0.488$ kg/m
(b) $\rho(1) = \left[\frac{dm}{dx}\right]_{x=1} = \left[\frac{1}{2\sqrt{x}}\right]_{x=1} = \frac{1}{2}$ kg/m.

5. $m(x) = x+\frac{1}{2}x^2$; $\rho(6) = \left[\frac{dm}{dx}\right]_{x=6} = \left[(1+x)\right]_{x=6} = 7$ g/cm

6. $n = 1000+180t+25t^2+3t^3$
Growth rate $\left[\frac{dn}{dt}\right]_{x=3} = \left[(180+50t+9t^2\right]_{x=3} = 180+50(3)+9(3)^2$
$= 411$ bacteria/hour.

7. $\beta = -\frac{1}{V}\frac{dV}{dP}$, $V = \frac{5.3}{P}$
$\frac{dV}{dP} = -\frac{5.3}{P^2}$, therefore $\beta = -\frac{1}{\frac{5.3}{P}}\times -\frac{5.3}{P^2} = \frac{1}{P}$, so when $P = 40$ kPa, $\beta = \frac{1}{40}$ m^3/kPa/m^3.

8.



The rate of reaction after 2 minutes equals the negative slope of the tangent.
$\frac{\Delta[N_2O_5]}{\Delta t} = -\frac{[N_2O_5](t_2)-[N_2O_5](t_1)}{t_2-t_1}$
$= -\frac{0.02-0.140}{4-0} = -0.03$ mol/L/min

9. $v = \frac{P}{4\eta L}(R^2 - r^2)$, $P = 4000$ dynes/cm^2, $L = 2$ cm, $R = 0.08$ cm, $\eta = 0.027$

Rate of change of v with respect to r when $r = 0.005$ cm

$$\frac{dv}{dr} = -\frac{rP}{2\eta L}, \text{ so } \left[\frac{dv}{dr}\right]_{r=0.005} = -\frac{0.005(4000)}{2(0.027)(2)} = -185.2 \text{ cm/s/cm}$$

Exercise 3.4

1. $C(x) = 55\,000 + 23x + 0.012x^2$

(a) $C'(x) = 23 + 0.024\ x$

(b) $C'(100) = 23 + 0.024(100) =$ **\$25.40/item**

(c) $C(101) - C(100) = 55\,000 + 23(101) + 0.012(101)^2 - [55\,000 + 23(100) + 0.012(100)^2)]$

$= 2445.41 - 2420.00 = \$25.41$

2. $C(x) = 1500 + \frac{x}{10} + \frac{x^2}{1000}$

(a) $C'(x) = \frac{1}{10} + \frac{x}{500}$

(b) $C'(800) = \frac{1}{10} + \frac{800}{500} =$ **\$1.70/unit**

(c) $C(801) - C(800) = 1500 - \frac{801}{10} + \frac{(801)^2}{1000} - [1500 + \frac{800}{10} + \frac{(800)^2}{1000}] = 2221.70 - 2220.00$

$= \$1.70$

3. $R(x) = 8000x - 0.02x^3$

(a) $R'(x) = 8000 - 0.06x^2$

(b) $R'(300) = 8000 - 0.06(300)^2 = -$ **\$2600/unit**

(c) $R(301) - R(300) = 8000(301) - 0.02(301)^3 - [8000(300) - 0.02(300)^3]$

$= 1\,862\,582 - 1\,860\,000 = \$2581.98$

4. $C(x) = 23\,000 + 0.24x + 0.0001x^2$, $R(x) = 0.98x - 0.0002x^2$

(a) $P(x) = R(x) - C(x) = 0.98x - 0.0002x^2 - 23\,000 - 0.24x - 0.0001x^2$

$= 0.74x - 0.0003x^2 - 23\,000$

(b) $P'(x) = 0.74 - 0.0006x$

(c) $P'(1000) = 0.74 - 0.0006(1000) =$ **\$0.14/pen**

(d) $P(1001) - P(1000)$

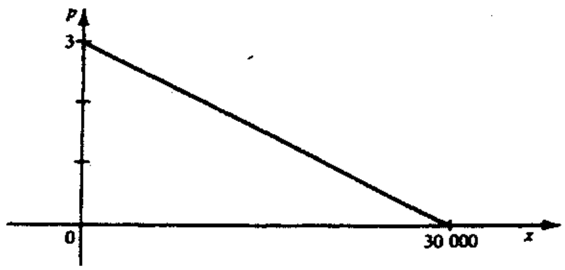
$= 0.74(1001) - 0.0003(1001)^2 - 23\,000 - [0.74(1000) - 0.0003(1000)^2 - 23\,000]$

$= -22\,559.8603 + 22\,560.0000 = \$0.1397$

Exercise 3.4

5. $p = \frac{30\,000 - x}{10\,000}$, $C(x) = 6000 + 0.8x$

(a) $p = \frac{30\,000 - x}{10\,000}$



(b)

p	0	\$0.50	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00
x	30000	25000	20000	15000	10000	5000	0

(c) $R(x) = xp(x) = \frac{1}{10\,000}(30\,000x - x^2)$ (d) $R'(x) = \frac{1}{10\,000}(30\,000 - 2x)$

(e) $R'(1000) \doteq \frac{1}{10\,000}(30\,000 - 2(1000)) = \frac{28\,000}{10\,000} = \$2.80$

(f) $P(x) = R(x) - C(x) = \frac{1}{10\,000}(30\,000x - x^2) - 600 - 0.8x = 2.2x - \frac{x^2}{10000} - 6000$

(g) $P'(x) = 2.2 - \frac{x}{5000}$ (h) $P'(10\,000) = 2.2 - \frac{10\,000}{5000} = \$0.20$

6. $C(x) = 82\,000 + 23x + 0.001x^2$, $p = 100 - 0.01x$

(a) $C'(x) = 23 + 0.002x$

(b) $R(x) = xp(x) = 100x - 0.01x^2$, so $R'(x) = 100 - 0.02x$

(c) $P'(x) = R'(x) - C'(x) = 100 - 0.02x - 23 - 0.002x = 77 - 0.022x$

(d) $P'(5) = 77 - 0.022(50) = \$75.90$

Exercise 3.5

1. $xy^2 = 12$, $\frac{dy}{dt} = 6$, find $\frac{dx}{dt}$ when $y = 2$

$2xy\frac{dy}{dt} + y^2\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{2x}{y}\frac{dy}{dt}$, but $x = \frac{12}{y^2}$, so $\frac{dx}{dt} = -\frac{24}{y^3}\frac{dy}{dt} = -\frac{24}{2^3}(6) = -18$

2. $x^3 + y^3 = 9$, $\frac{dx}{dt} = 4$, find $\frac{dy}{dt}$ when $x = 2$

$3x^2\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x^2}{y^2}\frac{dx}{dt}$, but $y^2 = (9 - x^3)^{\frac{2}{3}}$, so $\frac{dy}{dt} = -\frac{x^2}{\sqrt[3]{(9-x^3)^2}}\frac{dx}{dt}$

$= -\frac{2^2}{\sqrt[3]{(9-2^3)^2}}(4) = -16$

3. Let x be the length of the side of the square, let A be the area of the square, and let t be time in muinutes.

$\frac{dx}{dt} = 0.8$ m/min. We want the rate of increase of the area when $x = 3$ m.

$A = x^2 \Rightarrow \frac{dA}{dt} = 2x\frac{dx}{dt} = 2(3)(0.8) = 4.8$

The area is increasing at a rate of 4.8 m^2/min.

4. Let x be the edge length of the cube, let V be the volume of the cube, and let t be time in seconds.

$\frac{dV}{dt} = 144$ cm^3/s. We want the rate of increase of the edge length when $x = 4$ cm.

$V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{3x^2}\frac{dV}{dt} = \frac{1}{3(4)^2}(144) = 3$

The edge length is increasing at a rate of 3 cm/s.

5. Let r be the radius of the circle, let A be the area of the wave, let t be time in seconds. $\frac{dr}{dt} = 25$ cm/s. We want the rate of increase of the area after 4 s.

$A = \pi r^2 \Rightarrow \frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt} = 2\pi r\frac{dr}{dt}$. After 4 s, $r = 4(25) = 100$ cm, so $\frac{dA}{dt} =$ $2\pi(100)(25) = 5000\pi$. The area is increasing at a rate of $5000\pi \doteq 15700$ cm^2/s.

6. Let V be the volume of the balloon, let r be the radius, let t be time in minutes.

$\frac{dV}{dt} = 8$ m^3/min. We want the rate of increase of the radius when the diameter is 2 m.

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{dV}{dt}\left(\frac{1}{4\pi r^2}\right) = (8)\left(\frac{1}{4\pi(1)^2}\right) = \frac{2}{\pi}$

The radius of the balloon is increasing at $\frac{2}{\pi} \doteq 0.64$ m/min.

7. Let S be the surface area of the snowball, let r be its radius, let t be time in minutes.

$\frac{dS}{dt} = -0.5\ cm^2/min$. We want the rate of decrease of the radius when $r = 4$ cm.

$S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{8\pi r}\frac{dS}{dt} = \frac{1}{8\pi(4)}(-0.5) = -\frac{1}{64\pi}$

The radius is decreasing at a rate of $\frac{1}{64\pi} \doteq 0.005$ cm/min.

8. Let x be the side of the triangle, let A be its area, let t be time in seconds.

$\frac{dx}{dt} = -2$ cm/s. We want the rate of decrease of the area when $A = 100\ cm^2$.

$A = \frac{1}{2}(\text{base})(\text{heigth}) = \frac{1}{2}(x)(\frac{\sqrt{3}}{2}x) = \frac{\sqrt{3}}{4}x^2 \Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}x\frac{dx}{dt}$, but $x = \frac{2\sqrt{A}}{\sqrt[4]{3}}$, so

$\frac{dA}{dt} = \frac{\sqrt{3}}{2}\left(\frac{2\sqrt{A}}{\sqrt[4]{3}}\right)\frac{dx}{dt} = \frac{\sqrt{3}}{2}\left(\frac{2\sqrt{100}}{\sqrt[4]{3}}\right)(-2) = -\frac{20}{\sqrt{3}}$

The area is decreasing at a rate of $\frac{20}{\sqrt{3}} \doteq 11.5\ cm^2/s$.

9. Let A be the area of the triangle, let b be its base, let h be the altitude, let t be time in minutes.

$\frac{dA}{dt} = 4\ cm^2/min$, $\frac{db}{dt} = 1$ cm/min. We want the rate of increase of the altitude when $h = 20$ cm and $A = 80\ cm^2$.

$A = \frac{1}{2}bh \Rightarrow \frac{dA}{dt} = \frac{1}{2}b\frac{dh}{dt} + \frac{1}{2}h\frac{db}{dt} \Rightarrow \frac{dh}{dt} = \frac{2}{b}\frac{dA}{dt} - \frac{h}{b}\frac{db}{dt}$, but $b = \frac{2A}{h}$, so

$\frac{dh}{dt} = \frac{2}{\frac{2A}{h}}\frac{dA}{dt} - \frac{h}{\frac{2A}{h}}\frac{db}{dt} = \frac{2}{\frac{2(80)}{20}}(4) - \frac{20}{\frac{2(80)}{20}}(1) = -1.5$

So the altitude is decreasing at a rate of 1.5 cm/min.

10. Let y be the length of the shadow, let x be the distance from the man to the lamp, let t be time in seconds.

$\frac{dx}{dt} = 1.5$ m/s. We want the rate of increase of the length of the shadow when $x = 10$ m.

$\frac{y}{2} = \frac{x+y}{5} \Rightarrow 5y = 2x + 2y \Rightarrow y = \frac{2}{3}x \Rightarrow \frac{dy}{dt} = \frac{2}{3}\frac{dx}{dt} = \frac{2}{3}(1.5) = 1$

So the length of the shadow is increasing at a rate of 1 m/s.

11. Let x be the distance from the bottom of the ladder to the wall, let y be the distance from the top of the ladder to the ground, let t be time in seconds.

$\frac{dx}{dt} = 30$ cm/s $= \frac{3}{10}$ m/s. We want the rate of decrease of the distance from the top of the ladder to the ground when $x = 2$m.

$x^2 + y^2 = 4^2 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{dx}{dt}(-\frac{x}{y})$, but $y = \sqrt{16 - x^2}$, so

$\frac{dy}{dt} = \frac{dx}{dt}\left(-\frac{x}{\sqrt{16-x^2}}\right) = \frac{3}{10}\left(-\frac{2}{\sqrt{16-2^2}}\right) = -\frac{\sqrt{3}}{10}$

So the ladder is sliding down the wall at a rate of $\frac{\sqrt{3}}{10} \doteq 0.17$ m/s.

12. Let D be the distance from Dave's car to the intersection, let J be the distance from Joe's car to the intersection, let R be the distance between the two cars, let t be time in hours.

$\frac{dD}{dt} = -70$ km/h, $\frac{dJ}{dt} = -60$ km/h. We want the rate of decrease of the distance between the two cars when $D = 0.3$ km, $J = 0.4$ km.

$R = \sqrt{D^2+J^2} \Rightarrow \frac{dR}{dt} = \frac{1}{2\sqrt{D^2+J^2}}(2D\frac{dD}{dt} + 2J\frac{dJ}{dt})$

$= \frac{1}{2\sqrt{(0.4)^2+(0.3)^2}}[2(0.3)(-70)+2(-60)(0.4)] = -90$

So the distance between the cars is decreasing at a rate of 90 km/h.

13. Let A be the distance that ship A has travelled north, let B be the distance that ship B has travelled east, let R be the distance between the two ships, let t be time in hours.

$\frac{dA}{dt} = 30$ km/h, $\frac{dB}{dt} = 40$ km/h. We want the rate of change of the distance between the two ships at 3:00 P.M. ($t=2$ h).

$R^2 = (80-A)^2+B^2 = 6400-160A+A^2+B^2 \Rightarrow 2R\frac{dR}{dt} = -160\frac{dA}{dt} + 2A\frac{dA}{dt} + 2B\frac{dB}{dt}$

$\Rightarrow \frac{dR}{dt} = \frac{1}{2\sqrt{(80-A)^2+B^2}}\left(-160\frac{dA}{dt} + 2A\frac{dA}{dt} + 2B\frac{dB}{dt}\right)$, but $A = t\frac{dA}{dt} = 2(30) = 60$ and

$B = t\frac{dB}{dt} = 2(40) = 80$, so $\frac{dR}{dt} = \frac{1}{2\sqrt{(80-60)^2+(80)^2}}\left(-160(30)+2(60)(3)+2(80)(4)\right)$

$= \frac{130\sqrt{17}}{17}$

So the distance between the two ships is increasing at $\frac{130\sqrt{17}}{17} \doteq 31.5$ km/h.

14. Let h be the height of the skier above the water, let x be her horizontal distance from the beginning of the ramp, let R be the distance along the ramp that she has travelled, let t be time in seconds.

$\frac{dR}{dt} = 12$ m/s. We want the rate of change of her height when $x = 5$ m, $h = 1$ m.

$R^2 = x^2+h^2$, but $\frac{h}{x} = \frac{1}{5} \Rightarrow x = 5h$, so $R^2 = (5h)^2+h^2 = 26h^2 \Rightarrow 2R\frac{dR}{dt} = 52h\frac{dh}{dt}$

$$\Rightarrow \frac{dh}{dt} = \frac{\sqrt{x^2+h^2}}{26h}\frac{dR}{dt} = \frac{\sqrt{5^2+1^2}}{26(1)}(12) = \frac{6\sqrt{26}}{13}$$

So the skier is rising at a rate of $\frac{6\sqrt{26}}{13} \doteq 2.35$ m/s.

15. Let x be the horizontal distance from the plane to the town, let y be the altitude of the plane, let z be the distance from the plane to the town, let t be time in hours.

$\frac{dx}{dt} = 600$ km/h. We want the rate of increase of the distance from the plane to the town when $x = 20$ km, $y = 10$ km.

$$z^2 = x^2+y^2 \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}\text{ , but } \frac{dy}{dt} = 0 \text{ , so } \frac{dz}{dt} = \frac{1}{2\sqrt{x^2+y^2}}\left(2x\frac{dx}{dt}\right)$$

$$\Rightarrow \frac{dz}{dt} = \frac{1}{2\sqrt{(20)^2+(10)^2}}(2)(20)(600) = 240\sqrt{5}$$

So the distance between the plane and the town is increasing at a rate of $240\sqrt{5} \doteq 537$ km/h.

16. Let V be the volume of the trough, let h be the height of the water, let w be the width of the water, let t be time in minutes.

$\frac{dV}{dt} = 0.4$ m^3/min. We want the rate of increase of the height when $h = 40$ cm $= 0.4$ m.

$$V = \tfrac{1}{2}10wh \text{ , but } \frac{w}{h} = \frac{1}{\frac{1}{2}} \Rightarrow w = 2h, \text{ so } V = \tfrac{1}{2}10(2h)h - 10h^2 \Rightarrow \frac{dV}{dt} = 20h\frac{dh}{dt} \Rightarrow$$

$$\frac{dh}{dt} = \frac{1}{20h}\frac{dV}{dt} = \frac{1}{20(0.4)}(0.4) = \frac{1}{20}$$

So the water level is rising at a rate of 0.05 m/min = 5 cm/min.

17. Let V be the volume of the cone, let h be its height, let r be the radius of the base, let t be time in minutes.

$\frac{dV}{dt} = 1.2$ m^3/min. We want the rate of increase of the height when $h = 3$m.

$$V = \tfrac{1}{3}\pi r^2 h \text{ , but } 2r = h \Rightarrow r = \tfrac{1}{2}h, \text{ so } V = \tfrac{1}{3}\pi(\tfrac{1}{2}h)^2 h = \tfrac{1}{12}\pi h^3 \Rightarrow \frac{dV}{dt} = \tfrac{1}{4}\pi h^2\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt} = \frac{4}{\pi 3^2}(1.2) = \frac{8}{15\pi}$$

So the height of the pile is increasing at a rate of $\frac{8}{15\pi} \doteq 0.17$ m/min.

Exercise 3.6

1. $x^3+2x+1=0$, $x_1=0$, find x_2:

$f(x)=x^3+2x+1$, $f'(x)=3x^2+2$, so $x_2=x_1-\dfrac{f(x_1)}{f'(x_1)}=0-\dfrac{0^3+2(0)+1}{3(0)^2+2}=0-\frac{1}{2}=-\frac{1}{2}$

2. $x^3+x^2+1=0$, $x_1=-1$, find x_2 and x_3:

$f(x)=x^3+x^2+1$, $f'(x)=3x^2+2x$, so $x_2=x_1-\dfrac{f(x_1)}{f'(x_1)}=-1-\dfrac{(-1)^3+(-1)^2+1}{3(-1)^2+2(-1)}$

$=-1-1=-2$

$x_3=x_2-\dfrac{f(x_2)}{f'(x_2)}=-2-\dfrac{(-2)^3+(-2)^2+1}{3(-2)^2+2(-1)}=-2+\frac{3}{8}=-\frac{13}{8}$

3. (a) $x_1=2$, find root of $x^3-x-2=0$ to six decimal places.

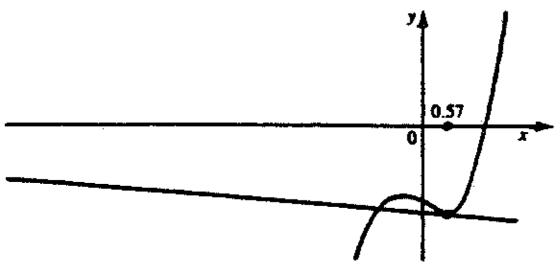
$f(x)=x^3-x-2$, $f'(x)=3x^2-1$, so $x_{n+1}=x_n-\dfrac{x_n^3-x_n-2}{3x_n^2-1}$

$x_1=2$	$x_3\doteq 1.530\,392$	$x_5\doteq 1.521\,380$
$x_2\doteq 1.636\,364$	$x_4\doteq 1.521\,441$	$x_6\doteq 1.521\,380$

(b) Using $x_1=1$ gives $x_2=2$, so x_3 through x_5 are the same as x_2 through x_4 from part (a), and $x_6=x_7\doteq 1.521\,380$.

(c) Using $x_1=0.57$,

$x_2\doteq -93.691\,146$	$x_9\doteq -5.518\,062$	$x_{16}\doteq -1.379\,517$
$x_3\doteq -62.463\,060$	$x_{10}\doteq -3.697\,289$	$x_{17}\doteq -0.690\,272$
$x_4\doteq -41.645\,427$	$x_{11}\doteq -2.476\,478$	$x_{18}\doteq 3.125\,594$
$x_5\doteq -27.768\,571$	$x_{12}\doteq -1.630\,925$	$x_{19}\doteq 2.227\,990$
$x_6\doteq -18.519\,522$	$x_{13}\doteq -0.956\,517$	$x_{20}\doteq 1.736\,217$
$x_7\doteq -12.356\,413$	$x_{14}\doteq 0.143\,122$	$x_{21}\doteq 1.550\,036$
$x_8\doteq -8.251\,257$	$x_{15}\doteq -2.137\,198$	$x_{22}\doteq 1.521\,991$
		$x_{23}\doteq 1.521\,380$



4. (a) $f(x)=x^4-x^2+x-5$, $1<x<2$

so $f'(x)=4x^3-2x+1$ which gives $x_{n+1}=x_n-\dfrac{x_n^4-x_n^2+x_n-5}{4x_n^3-2x_n+1}$

Guess $x_1=1.5$. Then

$x_2 \doteq 1.559783$ $\quad$ $x_4 \doteq 1.556250$

$x_3 \doteq 1.556263$ $\quad$ $x_5 \doteq 1.556250$

(b) $f(x) = x^3 - x^2 + 2x - 9,\ 2 < x < 3$

so $f'(x) = 3x^2 - 2x + 2$, which gives $x_{n+1} = x_n - \dfrac{x_n^3 - x_n^2 + 2x_n - 9}{3x_n^2 - 2x_n + 2}$

Guess $x_1 = 2$

$x_2 = 2.100000$ $\quad$ $x_4 \doteq 2.095366$

$x_3 \doteq 2.095376$ $\quad$ $x_5 \doteq 2.095366$

(c) $x^6 = \sqrt{x+7} \Rightarrow x^{12} = x + 7 \Rightarrow f(x) = x^{12} - x - 7,\ 1 < x < 2$

so $f'(x) = 12x^{11} - 1$, which gives $x_{n+1} = x_n - \dfrac{x_n^{12} - x_n - 7}{12x_n^{11} - 1}$

Guess $x_1 = 1.5$

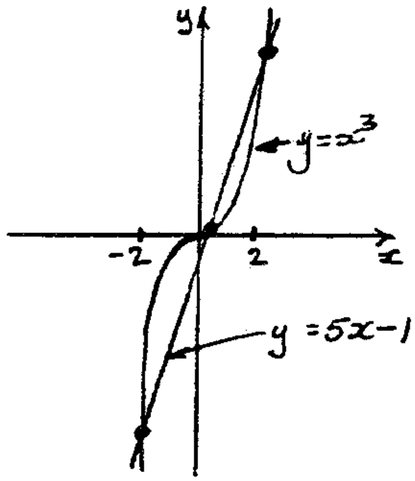
$x_2 \doteq 1.383076$ $\quad$ $x_6 \doteq 1.191629$

$x_3 \doteq 1.287315$ $\quad$ $x_7 \doteq 1.191554$

$x_4 \doteq 1.222628$ $\quad$ $x_8 \doteq 1.191554$

$x_5 \doteq 1.195587$

5. (a) $x^3 - 5x + 1 = 0$; To find out how many roots there are, draw $y = x^3$ and $y = 5x - 1$ on the same graph; the number of intersection points gives the number of roots, and the approximate x-co-ordinates of these intersection points will give the initial values to use with Newton's method.



From the graph, it can be seen that the approximate x-co-ordinates of the intersection points are $x = -2$, $x = 0$, and $x = 2$.

If $f(x) = x^3 - 5x + 1$, then $f'(x) = 3x^2 - 5$ so

$$x_{n+1} = x_n - \frac{x_n^3 - 5x_n + 1}{3x_n^2 - 5}$$

Guess $x_1 = -2$ $\quad$ $x_2 \doteq -2.428571$

$x_3 \doteq -2.335554$ $\quad$ $x_4 \doteq -2.330077$

$x_5 \doteq -2.330059$ $\quad$ $x_6 \doteq -2.330059$

Guess $x_1 = 0$ $\quad$ $x_2 = 0.200000$

$x_3 \doteq 0.201639$ $\quad$ $x_4 \doteq 0.201640$

$x_5 \doteq 0.201640$

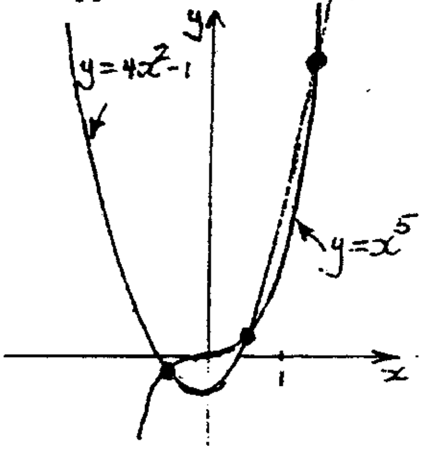
Guess $x_1 = 2$ $\quad$ $x_2 \doteq 2.142857$

$x_3 \doteq 2.128571$ $\quad$ $x_4 \doteq 2.128419$

$x_5 \doteq 2.128419$

Correct to 6 decimal places, the roots are -2.330059, 0.201640, and 2.128419.

(b) $x^5 = 4x^2 - 1$; Using the same procedure as above, draw $y = x^6$ and $y = 4x^2 - 1$ to give approximate root values.



From the graph, we can see that the approximate x co-ordinates are $x = -0.5$, $x = 0.5$, $x = 1.5$.

If $f(x) = x^5 - 4x^2 + 1$, then $f'(x) = 5x^4 - 8x$,

so $x_{n+1} = x_n - \dfrac{x_n^5 - 4x_n^2 + 1}{5x_n^4 - 8x_n}$

Guess $x_1 = -0.5$

$x_2 \doteq -0.492754$ $\qquad$ $x_4 \doteq -0.492689$

$x_3 \doteq -0.492689$

Guess $x_1 = 1.5$

$x_2 = 1.530516$ $\qquad$ $x_4 = 1.528643$

$x_3 = 1.528650$ $\qquad$ $x_5 = 1.528643$

Guess $x_1 = 0.5$

$x_2 \doteq 0.508475$ $\qquad$ $x_4 \doteq 0.508422$

$x_3 \doteq 0.508422$

6. (a) Derive $x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n})$ using Newton's method.

Let $x^2 - a = 0$ (this gives $x = \sqrt{a}$) , so $f(x) = x^2 - a$ which gives $f'(x) = 2x$ so

$$x_{n+1} = x_n - \frac{x_n^2 - a}{2x_n} = \frac{2x_n^2 - x_n^2 + a}{2x_n} = \frac{x_n^2 + a}{2x_n} = \frac{1}{2}(x_n + \frac{a}{x_n})$$

(b) Compute $\sqrt{17.2}$ using $x_{n+1} = \frac{1}{2}(x_n + \frac{17.2}{x_n})$ to six decimal places.

Guess $x_1 = 4$

$x_2 = 4.150000$ $\qquad$ $x_4 \doteq 4.147288$

$x_3 \doteq 4.147289$ $\qquad$ $x_5 \doteq 4.147288$

So $\sqrt{17.2} \doteq 4.147288$ to six decimal places.

7. (a) Find $\sqrt[5]{28}$ to six decimal places.

This is equivalent to solving $x^5 - 28 = 0$, so let $f(x) = x^5 - 28$ which gives

$f'(x) = 5x^4$, so $x_{n+1} = x_n - \dfrac{x_n^5 - 28}{5x_n^4}$

Guess $x_1 = 2$

$x_2 = 1.950000$ $\qquad$ $x_4 \doteq 1.947294$

$x_3 \doteq 1.947302$ $\qquad$ $x_5 \doteq 1.947294$

So $\sqrt[5]{28} \doteq 1.947294$ to six decimal places.

(b) Find $\sqrt[8]{1.23}$ to six decimal places.

This is equivalent to solving $x^8 - 1.23 = 0$, so let $f(x) = x^8 - 1.23$ which gives

$f'(x) = 8x^7$, so $x_{n+1} = x_n - \dfrac{x_n^8 - 1.23}{8x_n^7}$

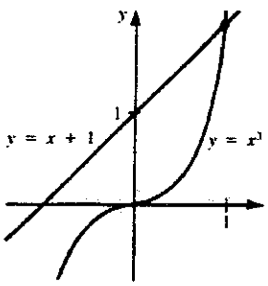
Guess $x_1 = 1$

$x_2 = 1.028750$ $\qquad$ $x_4 = 1.026214$

$x_3 = 1.026236$ $\qquad$ $x_5 = 1.026214$

So $\sqrt[8]{1.23} \doteq 1.026214$ to six decimal places.

8. (a) $y = x^3$, $y = x+1$



The x-coordinate of the intersection point satisfies $x^3 = x+1$, so it is a root of $x^3 - x - 1 = 0$.

Let $f(x) = x^3 - x - 1$, then $f'(x) = 3x^2 - 1$,

so $x_{n+1} = x_n - \dfrac{x_n^3 - x_n - 1}{3x_n^2 - 1}$

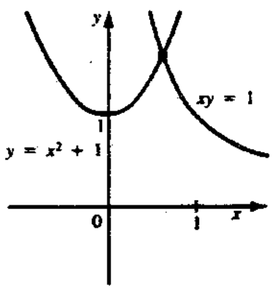
From the graph, the x co-ordinate of the intersection point is approximately $x = 1.5$. So $x_1 = 1.5$

$x_2 \doteq 1.347826$ $\qquad$ $x_4 \doteq 1.324718$

$x_3 \doteq 1.325200$ $\qquad$ $x_5 \doteq 1.324718$

So, to six decimal places, the x co-ordinate is $x = 1.324718$. Using $y = x+1$, the y co-ordinate is $y \doteq 2.324718$. So the point of intersection is (1.324718, 2.324718) to six decimal places.

(b) $y = x^2+1$, $xy = 1 \Rightarrow y = \frac{1}{x}$



The x-coordinate satisfies $x^2+1 = \frac{1}{x}$, so it is a root of $x^2 - \frac{1}{x} + 1 = 0$.

Let $f(x) = x^2 - \frac{1}{x} + 1$, then $f'(x) = 2x + \frac{1}{x^2}$,

so $x_{n+1} = x_n - \dfrac{x_n^2 - \frac{1}{x_n} + 1}{2x_n + \frac{1}{x_n^2}}$

From the graph, the x co-ordinate of the intersection point is approximately $x = 1$.

So $x_1 = 1$

$x_2 \doteq 0.666667$ $\qquad$ $x_4 = 0.682328$

$x_3 \doteq 0.682171$ $\qquad$ $x_5 = 0.682328$

So, to six decimal places, the x-coordinate is $x = 0.682328$. Using $y = \frac{1}{x}$, the y-co-ordinate is $y \doteq \dfrac{1}{0.682328} \doteq 1.465571$. So the point of intersection is (0.682328, 1.465571) to six decimal places.

Review Exercise 3.7

1. $s(t) = 2t^3 + 4t^2 - t$

(a) $v(t) = \frac{ds}{dt} = 6t^2 + 8t - 1$

$a(t) = \frac{d^2s}{dt^2} = 12t + 8$

(b) $v(4) = 6(4)^2 + 8(4) - 1 = 127$ m/s

$a(4) = 12(4) + 8 = 56$ m/s^2

2. $s(t) = t^3 - 12t^2 + 45t + 3$

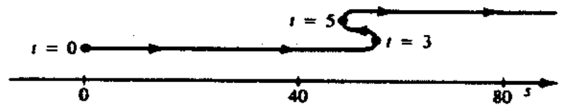
(a) $v(t) = 3t^2 - 24t + 45 = 0 \Rightarrow (t-5)(t-3) = 0 \Rightarrow t = 3,\ t = 5$; So the particle is at rest when $t = 3$ s and when $t = 5$ s.

(b) $v(t) > 0 \Rightarrow (t-5)(t-3) > 0 \Rightarrow t-5 > 0$ and $t-3 > 0$ or $t-5 < 0$ and $t-3 < 0 \Rightarrow$ $t > 5$ and $t > 3$ or $t < 5$ and $t < 3 \Rightarrow t > 5$ or $t < 3$ $(t \geq 0)$. So the velocity is positive when $t > 5$ s or $0 \leq t < 3$ s, and the velocity is negative when $3 < t < 5$ s.

(c) $a(t) = 6t - 24 > 0$ when $t > 4$. So the acceleration is positive when $t > 4$ s and is negative when $0 \leq t < 4$ s.

(d) $a(t) = 6t - 24 = 0$ when $t = 3$; $v(3) = 3(3)^3 - 24(3) + 45 = -3$ m/s.

(e)



(f) To find the total distance travelled after 8 s, the trip must be divided into segments wherever the velocity changes sign.
For $0 \leq t < 3$, $d = s(3) - s(0)$, for $3 < t < 5$, $d = s(3) - s(5)$, for $5 < t < 8$, $d = s(8) - s(5)$.
So the total distance travelled $D = s(3) - s(0) + s(3) - s(5) + s(8) - s(5)$
$= 2[s(3)] - 2[s(5)] - s(0) + s(8) = 2(57) - 2(53) - 3 + 107 = 112$ m.

3. $h(t) = 65t - 0.83t^2$

(a) $v(t) = 65 - 1.66t$, $v(1) = 65 - 1.66 = 63.34$ m/s

(b) $a(t) = -1.66$, $a(1) = -1.66$ m/s^2

(c) $h = 0 \Rightarrow 65t - 0.83t^2 = 0 \Rightarrow t(65 - 0.83t) = 0 \Rightarrow t = 0,\ t = \frac{65}{0.83} \doteq 78.3$. So the ball hits the moon after 78.3 s.

(d) $v(78.31) = 65 - 1.66(78.31) = -65$ m/s. The ball hits the moon at 65 m/s.

4. Rate of change of area of a square with respect to length of side L when $L = 5$.
$A = L^2 \Rightarrow \frac{dA}{dL} = 2L = 10$ units^2/unit

5. $m = 2+x+\frac{1}{2}x^2$

(a) For $x=2$ m to $x=2.1$ m, density $= \dfrac{m(2.1)-m(2)}{2.1-2} = \dfrac{2+2.1+\frac{1}{2}(2.1)^2-[2+2+\frac{1}{2}(2)^2]}{0.1}$

$= 3.05$ kg/m

(b) linear density $\rho(x) = \dfrac{dm}{dx} = 1+x$, $\rho(2) = 3$ kg/m

6. $C(x) = 19\,000+16.2x+0.06x^2$

(a) $C'(x) = 16.2+0.12x$

(b) $C'(200) = 16.2+0.12(200) = \$40.20$/unit

(c) $C(201)-C(200) = 19\,000+16.2(201)+0.06(201)^2-[19\,000+16.2(200)+0.06(200)^2]$

$= 24\,680.26-24\,640 = \$40.26$/unit

7. $C(x) = 12\,500+1.08x$, $p = \dfrac{20\,000-x}{1000}$

(a) $C'(x) = 1.08$

(b) $R(x) = xp(x) = 20x-0.001x^2$

(c) $R'(x) = 20-0.002x$

(d) $P(x) = R(x)-C(x) = 20x-0.001x^2-12\,500-1.08x = -0.001x^2+18.92x-12\,500$

(e) $P'(x) = -0.002x+18.92$

(f) $P'(8000) = -0.002(8000)+18.92 = \$2.92$/pizza

8. Let x be the width of the rectangle, let P be it's perimeter, let A be it's area, let t be time in minutes.

$\dfrac{dP}{dt} = 6$ cm/min. We want the rate of increase of the area when $P = 40$ cm.

$P = 6x \Rightarrow x = \dfrac{P}{6}$ (since length$=2\times$width), $A = 2x^2 = 2(\frac{P}{6})^2 = \dfrac{P^2}{18} \Rightarrow \dfrac{dA}{dt} = \dfrac{P}{9}\dfrac{dP}{dt}$

$= \frac{40}{9}(6) = \frac{80}{3}$

So the area is increasing at a rate of $\frac{80}{3} \doteq 26.7$ cm^2/min.

9. Let V be the volume of the gas, let P be the pressure, let t be time in minutes.

$\dfrac{dP}{dt} = 15$ kPa/min. We want the rate of decrease of the volume when $V = 480$ cm^3 and $P = 160$ kPa.

$PV = C$ (constant), so $\dfrac{dP}{dt}V+\dfrac{dV}{dt}P = 0 \Rightarrow \dfrac{dV}{dt} = -\dfrac{V}{P}\dfrac{dP}{dt} = -\dfrac{480}{160}(15) = -45$

So the volume is decreasing at a rate of 45 cm^3/min.

10. Let A be the distance that ship A travels, let B be the distance that ship B travels, let D be the distance between them, let t be time in hours.

$\dfrac{dA}{dt} = 40$ km/h, $\dfrac{dB}{dt} = 30$ km/h. We want the rate of increase of the distance between the two ships when $t = 3$ h.

$D^2 = (A+B)^2+50^2 \Rightarrow 2D\dfrac{dD}{dt} = 2(A+B)(\dfrac{dA}{dt}+\dfrac{dB}{dt})$, but $A = t\dfrac{dA}{dt}$ and $B = t\dfrac{dB}{dt} \Rightarrow$

$$\frac{dD}{dt} = \frac{t}{\sqrt{(t\frac{dA}{dt}+t\frac{dB}{dt})^2+50^2}}\left(\frac{dA}{dt}+\frac{dB}{dt}\right)^2 = \frac{3}{\sqrt{[(3)(40)+(3)(30)]^2+50^2}}\left(40+30\right)^2 = \frac{1470}{\sqrt{466}}$$

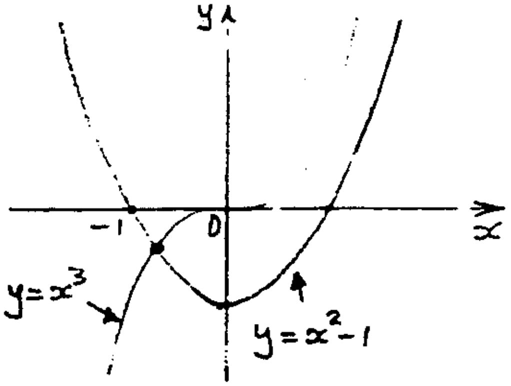
So the distance is increasing at a rate of $\frac{1470}{\sqrt{466}} \doteq 68.1$ km/h.

11. $x^4+x-1=0$, $x_1=1$, find x_2

$f(x)=x^4+x-1$, so $f'(x)=4x^3+1$, so $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{1^4+1-1}{4(1)^3+1} = 1-\frac{1}{5} = \frac{4}{5}$

12. $x^3-x^2+1=0$, find all roots to six decimal places.

To find the number of and approximate value of all roots, we sketch $y=x^3$ and $y=x^2-1$ on the same graph to find the number and approximate x value of all intersection points.



From the graph, we see that the only intersection is at $x \doteq -0.5$.

Let $f(x)=x^3-x^2+1$, then $f'(x)=3x^2-2x$,

so $x_{n+1} = x_n - \frac{x_n^3-x_n^2+1}{3x_n^2-2x_n}$

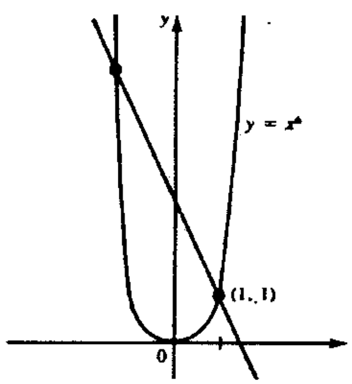
Guess $x_1 = -0.5$

$x_2 \doteq -0.857143$ $\quad x_5 \doteq -0.754878$

$x_3 \doteq -0.764137$ $\quad x_6 \doteq -0.754878$

$x_4 \doteq -0.754963$

13. (a)



(b) Find intersection points.

The x-coordinate of the intersection point satisfies $x^6=3-2x$, so it is a root of $x^6+2x-3=0$.

Let $f(x)=x^6+2x-3$, then $f'(x)=6x^5+2$,

so $x_{n+1} = x_n - \frac{x_n^6+2x_n-3}{6x_n^5+2}$

One intersection point is (1,1). The other appears to occur when $x \doteq -1$.

So guess $x_1 = -1$. Then

$x_2 = -2.000000$ $\quad x_6 \doteq -1.337747$

$x_3 = -1.700000$ $\quad x_7 \doteq -1.335398$

$x_4 \doteq -1.486786$ $\quad x_8 \doteq -1.335387$

$x_5 \doteq -1.370700$ $\quad x_9 \doteq -1.335387$

When $x \doteq -1.335387$, $y \doteq 3-2(-1.335387) \doteq 5.670774$. So the intersection points are (1,1) and $(-1.335387, -5.670774)$ correct to six decimal places.

Chapter 3 Test

1. $s(t) = t^3 - 6t^2 + 9t + 1$

(a) $v(t) = 3t^2 - 12t + 9$, $v(4) = 3(4)^2 - 12(4) + 9 = 9$ m/s.

(b) $a(t) = 6t - 12$, $a(4) = 6(4) - 12 = 12$ m/s^2

(c) $v(t) = 0 \Rightarrow 3t^2 - 12t + 9 = 0 \Rightarrow (t-1)(t-3) = 0 \Rightarrow t = 1, t = 3$. So the particle is at rest when $t = 1$ s, $t = 3$ s.

(d) $v(t) > 0 \Rightarrow (t-1)(t-3) > 0 \Rightarrow t - 1 > 0$ and $t - 3 > 0$ or $t - 1 < 0$ and $t - 3 < 0 \Rightarrow$ $t > 1$ and $t > 3$ or $t < 1$ and $t < 3 \Rightarrow t > 3$ or $t < 1$ $(t \geq 0)$. So the velocity is positive when $t > 3$ s or $0 \leq t < 1$ s.

(e) $a(t) = 6t - 12 = 0$ when $t = 2$ s, $v(2) = 3(2)^2 - 12(2) + 9 = -3$ m/s

(f) Total distance travelled in first 4 s.

$D = s(1) - s(0) + s(1) - s(3) + s(4) - s(3) = 2[s(1)] - s(0) - 2[s(3)] + s(4)$

$= 2(5) - 1 - 2(1) + 5 = 12$ m.

2. $C(x) = 87000 + 122x$, $p = \dfrac{600000 - x}{1000}$

(a) $C'(x) = 122$

(b) $R(x) = xp(x) = 600x - 0.001x^2$

(c) $R'(x) = 600 - 0.002x$

(d) $P(x) = 600x - 0.001x^2 - 87000 - 122x = -0.001x^2 + 478x - 87000$

e) $P'(x) = -0.002x + 478$

3. Let V be the volume of water in the cone, let h be the water's height, let r be its radius, let t be time in seconds.

$\frac{dV}{dt} = 2$ cm^3/s. We want the rate of increase of the height when $h = 6$ cm.

$V = \frac{1}{3}\pi r^2 h$, but $\frac{3}{8} = \frac{r}{h} \Rightarrow r = \frac{3h}{8}$, so $V = \frac{1}{3}\pi(\frac{3h}{8})^2 h = \frac{3}{64}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{9}{64}\pi h^2 \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{64}{9}\frac{1}{\pi h^2}\frac{dV}{dt} = \frac{64}{9}\frac{1}{\pi 6^2}(2) = \frac{32}{81\pi}$

So the height is increasing at a rate of $\frac{32}{81\pi} \doteq 0.13$ cm/s.

4. $x^5 = x + 2$, find the root to six decimal places.

Let $f(x) = x^5 - x - 2$, then $f'(x) = 5x^4 - 1$, so $x_{n+1} = x_n - \dfrac{x_n^5 - x_n - 2}{5x_n^4 - 1}$

Guess $x_1 = 1.25$

$x_2 \doteq 1.267689$

$x_3 \doteq 1.267169$

$x_4 \doteq 1.267168$

$x_5 \doteq 1.267168$

To six decimal places, the root is $x = 1.267168$.

Cumulative Review For Chapters 1 To 3

1. (a) $\lim_{x\to 2} \frac{2x^2+1}{3x^2-4} = \frac{2(2)^2+1}{3(2)^2-4} = \frac{9}{8}$

(b) $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4} = \lim_{x\to 2} \frac{(x-2)(x-1)}{(x+2)(x-2)} = \lim_{x\to 2} \frac{(x-1)}{(x+2)} = \frac{1}{4}$

(c) $\lim_{x\to -1} \frac{x^2+2x+1}{x^3+1} = \lim_{x\to -1} \frac{(x+1)^2}{(x+1)(x^2-x+1)} = \lim_{x\to -1} \frac{x+1}{x^2-x+1} = 0$

(d) $\lim_{h\to 0} \frac{(2+h)^3-8}{h} = \lim_{h\to 0} \frac{8+12h+6h^2+h^3-8}{h} = \lim_{h\to 0} \frac{12h+6h^2+h^3}{h}$

$= \lim_{h\to 0} (12+6h+h^2) = 12$

(e) $\lim_{h\to 0} \frac{\frac{1}{(3+h)^2}-\frac{1}{9}}{h} = \lim_{h\to 0} \frac{9-(3+h)^2}{9h(3+h)^2} = \lim_{h\to 0} \frac{9-9-6h-h^2}{9h(3+h)^2} = \lim_{h\to 0} \frac{-6-h}{9(3+h)^2}$

$= -\frac{2}{27}$

(f) $\lim_{x\to -2} \frac{x^2-x-6}{2x^2+5x+2} = \lim_{x\to -2} \frac{(x-3)(x+2)}{(2x+1)(x+2)} = \lim_{x\to -2} \frac{x-3}{2x+1} = \frac{5}{3}$

(g) $\lim_{x\to 2^+} \sqrt{x^2-x-2} = \lim_{x\to 2^+} \sqrt{(x-2)(x+1)} = \sqrt{\lim_{x\to 2^+}(x-2) \lim_{x\to 2^+}(x+1)} = 0$

(h) $\lim_{x\to -5^-} \frac{2x+10}{|x+5|} = \lim_{x\to -5^-} \frac{2(x+5)}{-(x+5)} = -2$, since $|x+5| = -(x+5)$ if $x<-5$

2. (a)

(i) Since $f(x) = -x-2$ if $x<-2$, $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-}(-x-2) = 0$

(ii) Since $f(x) = 1-x^2$ if $-2\le x\le 2$, $\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+}(1-x^2) = -3$

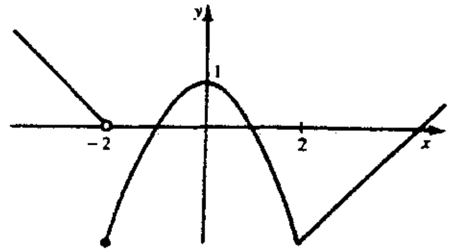
(iii) $\lim_{x\to -2} f(x)$ does not exist

(iv) Since $f(x) = 1-x^2$ if $-2\le x\le 2$, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(1-x^2) = -3$

(v) Since $f(x) = x-5$ if $x>2$, $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x-5) = -3$

(vi) $\lim_{x\to 2} f(x) = -3$

(b)



(c) f is discontinuous at $x = -2$

(d) f is not differentiable at $x = -2$, $x = 2$

3. (a) $\lim_{n\to\infty} \frac{1-2n^3}{n+n^3} = \lim_{n\to\infty} \frac{\frac{1}{n^3}-2}{\frac{1}{n^2}+1} = \frac{0-2}{0+1} = -2$

(b) $\lim_{n\to\infty} 3^{-n} = \lim_{n\to\infty} (\frac{1}{3^n}) = 0$

4. (a) $2-3+\frac{9}{2}-\frac{27}{4}+\ldots$; Geometric series with $a = 2$, $r = -\frac{3}{2}$. So the series diverges since $|r| > 1$.

(b) $3+2+\frac{4}{3}+\frac{8}{9}+\frac{16}{27}+\ldots$; Geometric series with $a = 3$, $r = \frac{2}{3}$. $S = \frac{3}{1-\frac{2}{3}} = 9$.

5. (a) $f(x) = 6-5x+3x^2$; $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{6-5(x+h)+3(x+h)^2-[6-5x+3x^2]}{h}$$

$$= \lim_{h\to 0} \frac{6-5x-5h+3x^2+6xh+3h^2-6+5x-3x^2}{h}$$

$$= \lim_{h\to 0} \frac{-5h+6xh+3h^2}{h} = \lim_{h\to 0} (-5+6x+3h) = 6x-5$$

(b) $f'(1) = 6-5 = 1$, so the slope of the tangent is 1. When $x = 1$, $y = 4$, so $y-4 = x-1 \Rightarrow y = x+3$ or $x-y+3 = 0$.

6. $g(x) = \sqrt{3-x}$, $g'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{3-x-h}-\sqrt{3-x}}{h} \times \frac{\sqrt{3-x-h}+\sqrt{3-x}}{\sqrt{3-x-h}+\sqrt{3-x}} = \lim_{h\to 0} \frac{3-x-h-3+x}{h\left(\sqrt{3-x-h}+\sqrt{3-x}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{3-x-h}+\sqrt{3-x}} = \frac{-1}{2\sqrt{3-x}}$$

7. (a) $f(x) = 12x^5 - \frac{1}{2}x^4 - 4x$; $f'(x) = 60x^4 - 2x^3 - 4$

(b) $f(x) = \dfrac{6}{x^2}$; $f'(x) = -\dfrac{12}{x^3}$

(c) $g(x) = \sqrt[3]{x}(2x + \frac{1}{x}) = 2x^{\frac{4}{3}} + x^{-\frac{2}{3}}$; $g'(x) = \frac{8}{3}x^{\frac{1}{3}} - \frac{2}{3}x^{-\frac{5}{3}}$

(d) $g(x) = \dfrac{x^2}{2x-3}$; $g'(x) = \dfrac{2x(2x-3) - 2x^2}{(2x-3)^2} = \dfrac{2x^2 - 6x}{(2x-3)^2} = \dfrac{2x(x-3)}{(2x-3)^2}$

(e) $f(t) = \sqrt{2t - t^3}$; $f'(t) = \dfrac{2 - 3t^2}{2\sqrt{2t - t^3}}$

(f) $f(y) = \left(\dfrac{2-y}{1+2y}\right)^4$; $f'(y) = 4\left(\dfrac{2-y}{1+2y}\right)^3\left(\dfrac{-(1+2y) - 2(2-y)}{(1+2y)^2}\right) = 4\left(\dfrac{2-y}{1+2y}\right)^3\left(\dfrac{-5}{(1+2y)^2}\right)$

$= -\dfrac{20(2-y)^3}{(1+2y)^5}$

(g) $y = (3x+5)(x^3-1)^3$; $y' = 3(x^3-1)^2(3x^2)(3x+5) + 3(x^3-1)^3$

$= 3(x^3-1)^2[3x^2(3x+5) + (x^3-1)] = 3(x^3-1)^2(10x^3 + 15x^2 - 1)$

(h) $y = \dfrac{1}{\sqrt[5]{x^5+1}} = (x^5+1)^{-\frac{1}{5}}$; $y' = -\frac{1}{5}(x^5+1)^{-\frac{6}{5}}(5x^4) = \dfrac{-x^4}{\sqrt[5]{(x^5+1)^6}}$

(i) $y = \dfrac{\sqrt{x} - x}{x^2} = x^{-\frac{3}{2}} - x^{-1}$; $y' = -\frac{3}{2}x^{-\frac{5}{2}} + x^{-2} = \dfrac{x^{-\frac{5}{2}}}{2}(-3 + 2\sqrt{x}) = \dfrac{2\sqrt{x} - 3}{2\sqrt{x^5}}$

(j) $y = \sqrt{\dfrac{x}{1+x^2}}$; $y' = \frac{1}{2}\sqrt{\dfrac{1+x^2}{x}} \times \dfrac{1+x^2-2x^2}{(1+x^2)^2} = \dfrac{1-x^2}{2\sqrt{x}\sqrt{(1+x^2)^3}}$

8. $f(x) = \dfrac{1}{\sqrt{x^2-1}}$; $f'(x) = \dfrac{-x}{\sqrt{(x^2-1)^3}}$, Domain of f = Domain of f' = $\{x \mid x^2 - 1 > 0\}$

$= \{x \mid x < -1 \text{ or } x > 1\}$

9. $x^4 + 2x^2y^3 + y^2 = 21$; $4x^3 + 6x^2y^2\dfrac{dy}{dx} + 4xy^3 + 2y\dfrac{dy}{dx} = 0 \Rightarrow$

$\dfrac{dy}{dx}(6x^2y^2 + 2y) = -(4x^3 + 4xy^3)$

$\dfrac{dy}{dx} = -\dfrac{2x^3 + 2xy^3}{3x^2y^2 + y}$

10. (a) $y = \dfrac{x+1}{x+2}$; $y' = \dfrac{x+2-(x+1)}{(x+2)^2} = \dfrac{1}{(x+2)^2}$, $y'' = \dfrac{-2}{(x+2)^3}$

(b) $x^2 - y^3 = 7$; $2x - 3x^2y' = 0 \Rightarrow y' = \dfrac{2x}{3y^2}$, $y'' = \dfrac{6y^2 - 2x(6yy')}{9y^4} = \dfrac{6y^2 - 12xy(\frac{2x}{3y^2})}{9y^4}$

$= \frac{6y^3-8x^2}{9y^5}$

11. (a) $y = \frac{2}{1+x^2}$, (1,1) ; $y' = \frac{-4x}{(1+x^2)^2}$, $y'(1) = \frac{-4}{4} = -1$, so $\frac{y-1}{x-1} = -1$
$\Rightarrow y-1 = -x+1 \Rightarrow y = -x+2$ or $x+y-2 = 0$.
(b) $x^2-y^2 = 3$, $(-2,-1)$; $2x-2yy' = 0 \Rightarrow y' = \frac{x}{y}$; at $(-2,-1)$, $y' = \frac{-2}{-1} = 2$, so
$\frac{y+1}{x+2} = 2 \Rightarrow y+1 = 2x+4 \Rightarrow y = 2x+3$ or $2x-y+3 = 0$.

12. $y = x^3-x$, tangents ∥ to $22x-2y+1 = 0 \Rightarrow y = 11x+\frac{1}{2}$
So $y' = 3x^2-1 = 11$ when $x = \pm 2$. At $x = 2$, $y = 6$; $x = -2$, $y = -6$.
$\frac{y-6}{x-2} = 11 \Rightarrow y-6 = 11x-22 \Rightarrow y = 11x-16$ or $11x-y-16 = 0$.
$\frac{y+6}{x+2} = 11 \Rightarrow y+6 = 11x+22 \Rightarrow y = 11x+16$ or $11x-y+16 = 0$.

13. $f(2) = -3$, $f'(2) = 10$, $f'(4) = 6$, $g(2) = 4$, $g'(2) = 1$.
(a) $(fg)'(2) = f(2)g'(2)+f'(2)g(2) = (-3)(1)+(10)(4) = 37$
(b) $(\frac{f}{g})'(2) = \frac{g(2)f'(2)-f(2)g'(2)}{[g(2)]^2} = \frac{(4)(10)-(-3)(1)}{(4)^2} = \frac{43}{16}$
(c) $(f \circ g)'(2) = f'(g(2))g'(2) = f'(4)(1) = 6$

14. (a) $F(x) = f(x^5)$; $F'(x) = 5x^4f'(x^5)$ (b) $F(x) = [f(x)]^5$; $F'(x) = 5[f(x)]^4f'(x)$
(c) $F(x) = x^5f(x)$; $F'(x) = 5x^4f(x)+x^5f'(x)$
(d) $F(x) = \sqrt{\frac{f(x)}{x}}$; $F'(x) = \frac{1}{2}\sqrt{\frac{x}{f(x)}}\left(\frac{xf'(x)-f(x)}{x^2}\right)$

15. $h(t) = 120-18t-4.9t^2$
(a) Average velocity
(i) $2 \le t \le 3$, $\bar{v} = \frac{h(3)-h(2)}{3-2} = 120-18(3)-4.9(3)^2-[120-18(2)-4.9(2)^2]$
$= 21.9-64.4 = -42.5$ m/s
(ii) $2 \le t \le 2.1$, $\bar{v} = \frac{h(2.1)-h(2)}{2.1-2} = \frac{120-18(2.1)-4.9(2.1)^2-64.4}{0.1} \doteq -38.09$ m/s
(iii) $2 \le t \le 2.01$, $\bar{v} = \frac{h(2.01)-h(2)}{2.01-2} = \frac{120-18(2.01)-4.9(2.01)^2-64.4}{0.01} \doteq -37.65$ m/s
(b) $v(t) = -18-9.8t$, $v(2) = -18-9.8(2) = -37.6$ m/s
(c) $a(t) = -9.8$, $a(2) = -9.8$ m/s^2

16. $s(t) = t^3 - 6t^2 + 9t + 5$

(a) $v(t) = 3t^2 - 12t + 9$, $v(2) = 3(2)^2 - 12(2) + 9 = -3$ m/s,

$v(4) = 3(4)^2 - 12(4) + 9 = 9$ m/s

(b) $a(t) = 6t - 12$, $a(2) = 6(2) - 12 = 0$ m/s^2, $a(4) = 6(4) - 12 = 12$ m/s^2

(c) $v(t) = 0 \Rightarrow 3t^2 - 12t + 9 = 0 \Rightarrow (t-3)(t-1) = 0 \Rightarrow t = 1$ s, $t = 3$ s.

(d) $v(t) > 0 \Rightarrow (t-3)(t-1) > 0 \Rightarrow t-3 > 0$ and $t-1 > 0$ or $t-3 < 0$ and $t-1 < 0 \Rightarrow$ $t > 3$ and $t > 1$ or $t < 3$ and $t < 1 \Rightarrow t > 3$ or $t < 1$ $(t \geq 0)$. So the velocity is positive when $t > 3$ s or $0 \leq t < 1$ s, and is negative when $1 < t < 3$ s.

(e) $a(t) > 0 \Rightarrow 6t - 12 > 0 \Rightarrow t > 2$. So the acceleration is positive when $t > 2$ s and is negative when $0 \leq t < 2$ s.

(f) Total distance travelled in first five seconds.

$D = s(1) - s(0) + s(1) - s(3) + s(5) - s(3) = 2[s(1)] - s(0) - 2[s(3)] + s(5)$

$= 2(9) - 5 - 2(5) + 25 = 28$ m.

17. (a) Let V be the volume of the balloon, let r be its radius.

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$; When $r = 0.5$ m, $\frac{dV}{dr} = 4\pi(0.5)^2 = \pi$ m^3/m.

(b) $\frac{dV}{dt} = 10$ m^3/min. We want the rate of increase of the radius when $r = 3$ m.

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt} = \frac{1}{4\pi(3)^2}(10) = \frac{5}{18\pi}$

So the radius is increasing at a rate of $\frac{5}{18\pi} \doteq 0.088$ m/min.

18. Let y be the length of the rope between the boat and the pulley, let x be the distance from the boat to the dock, let t be time in seconds.

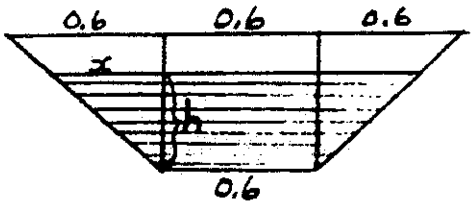
$\frac{dy}{dt} = -0.8$ m/s. We want the rate of decrease of the distance from the dock to the boat when $x = 10$ m.

$y^2 = x^2 + 1 \Rightarrow 2y\frac{dy}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt}$. When $x = 10$, $y^2 = 10^2 + 1 \Rightarrow y = \sqrt{101}$,

so $\frac{dx}{dt} = \frac{\sqrt{101}}{10}(-0.8) = -\frac{2\sqrt{101}}{25}$

So the boat is approaching the dock at a speed of $\frac{2\sqrt{101}}{25} \doteq 0.8$ m/s.

19.



Let V be the volume of water, let h be the height of the water, let x be half the width greater than 0.6 m, let t be time in minutes.

$\frac{dV}{dt} = 0.5$ m^3/min. We want the rate of increase of the height of the water when $h = 0.5$ m.

$\frac{x}{h} = \frac{.6}{.9} = \frac{2}{3}$, so $x = \frac{2}{3}h$.

The area of the isosceles trapezoid cross-section is the area of the rectangle plus the two triangles on the sides.

$A = 0.6h + 2(\frac{1}{2}xh) = 0.6h + xh = \frac{3}{5}h + \frac{2}{3}h^2$, $V = 6(\frac{3}{5}h + \frac{2}{3}h^2) = \frac{18}{5}h + 4h^2$

$\frac{dV}{dt} = \frac{18}{5}\frac{dh}{dt} + 8h\frac{dh}{dt}$, so $\frac{dh}{dt} = \frac{\frac{dV}{dt}}{3.6 + 8h} = \frac{0.5}{3.6 + 8(0.5)} = \frac{0.5}{7.6} = \frac{5}{76}$.

So the water level is rising at a rate of $\frac{5}{76}$ m/min $\doteq$ 6.6 cm/min.

20. Find root of $x^3 = 2x + 5$ to six decimal places.

Let $f(x) = x^3 - 2x - 5$, then $f'(x) = 3x^2 - 2$, so $x_{n+1} = x_n - \frac{x_n^3 - 2x_n - 5}{3x_n^2 - 2}$

Guess $x_1 = 2$

$x_2 \doteq 2.100000$

$x_3 \doteq 2.094568$

$x_4 \doteq 2.094551$

$x_5 \doteq 2.094551$

So the root is $x = 2.094551$ to six decimal places.