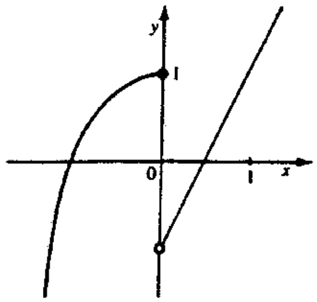
(b)



(c) f is discontinuous at 0

4. $s = 5t^2 - 6t + 14$

(a) $2 \le t \le 3$, $\bar{v} = \dfrac{s(3) - s(2)}{3-2} = 5(3)^2 - 6(3) + 14 - [5(2)^2 - 6(2) + 14] = 19$ m/s

(b) $v(2) = \lim_{h \to 0} \dfrac{s(2+h) - s(2)}{h} = \lim_{h \to 0} \dfrac{5(2+h)^2 - 6(2+h) + 14 - [5(2)^2 - 6(2) + 14]}{h}$

$= \lim_{h \to 0} \dfrac{20 + 20h + 5h^2 - 12 - 6h + 14 - 22}{h} = \lim_{h \to 0} \dfrac{14h + 5h^2}{h}$

$= \lim_{h \to 0} (14 + 5h) = 14$ m/s

5. $\lim_{n \to \infty} \left(\frac{1}{8^n} + \frac{6n-2}{2n-3}\right) = \lim_{n \to \infty} \left(\frac{1}{8^n}\right) + \lim_{n \to \infty} \frac{6n-2}{2n-3} = 0 + \lim_{n \to \infty} \dfrac{6 - \frac{2}{n}}{2 - \frac{3}{n}} = \frac{6-0}{2-0} = 3$

6. $12 - 9 + \frac{27}{4} - \frac{81}{16} + \dots$; $a = 12$, $r = -\frac{3}{4} \Rightarrow S = \dfrac{12}{1 + \frac{3}{4}} = \frac{48}{7}$

Review and Preview to Chapter 2

EXERCISE 1

1. (a) $f(x)=1-18x,\ x\in R$

(b) $g(x)=x^4-x^2+15x,\ x\in R$

(c) $h(x)=\sqrt{x-5},\ \{x \mid x-5\geq 0\}=\{x \mid x\geq 5\}$

(d) $F(x)=\sqrt[4]{-x},\ \{x \mid -x\geq 0\}=\{x \mid x\leq 0\}$

(e) $G(x)=\sqrt{1-x^2},\ \{x \mid 1-x^2\geq 0\}=\{x \mid x^2\leq 1\}=\{x \mid |x|\leq 1\}=\{x \mid -1\leq x\leq 1\}$

(f) $H(x)=\sqrt{x^2-2},\ \{x \mid x^2-2\geq 0\}=\{x \mid x^2\geq 2\}=\{x \mid |x|\geq\sqrt{2}\}$

$=\{x \mid x\geq\sqrt{2} \text{ or } x\leq -\sqrt{2}\}$

(g) $y=\dfrac{3+x}{3-x},\ \{x \mid 3-x\neq 0\}=\{x \mid x\neq 3\}$

(h) $y=\dfrac{x^2}{x^2+4x-5},\ \{x \mid x^2+4x-5\neq 0\}=\{x \mid (x+5)(x-1)\neq 0\}$

$=\{x \mid x\neq -5,\ x\neq 1\}$

(i) $y=\dfrac{1}{\sqrt{t^2+5}},\ \{t \mid t^2+5>0\}\Rightarrow t\in R$

(j) $y=\dfrac{t}{\sqrt{t^2-5t+6}},\ \{t \mid t^2-5t+6>0\}=\{t \mid (t-2)(t-3)>0\}=\{t \mid t<2 \text{ or } t>3\}$

(k) $f(x)=\sqrt{x}+\sqrt{4-x},\ \{x \mid x\geq 0 \text{ and } 4-x\geq 0\}=\{x \mid 0\leq x\leq 4\}$

(l) $f(x)=\sqrt{2-\sqrt{4-x}},\ \{x \mid 4-x\geq 0 \text{ and } \sqrt{4-x}\leq 2\}=\{x \mid x\leq 4 \text{ and } 4-x\leq 4\}$
$=\{x \mid 0\leq x\leq 4\}$

EXERCISE 2

1. (a) $f(x)=2x-1,\ g(x)=4-3x,$

$(f\circ g)(x)=f(g(x))=f(4-3x)=2(4-3x)-1=7-6x$

$(g\circ f)(x)=g(f(x))=g(2x-1)=4-3(2x-1)=7-6x$

$(f\circ f)(x)=f(f(x))=f(2x-1)=2(2x-1)-1=4x-3$

$(g\circ g)(x)=g(g(x))=g(4-3x)=4-3(4-3x)=9x-8$

(b) $f(x)=x^2,\ g(x)=x+1$

$(f\circ g)(x)=f(x+1)=(x+1)^2=x^2+2x+1$

$(g\circ f)(x)=g(x^2)=x^2+1$

$(f\circ f)(x)=f(x^2)=(x^2)^2=x^4$

$(g\circ g)(x)=g(x+1)+1=x+2$

(c) $f(x) = 1 - x^2$, $g(x) = 5$

$(f \circ g)(x) = f(5) = 1 - 5^2 = -24$ $\quad$ $(g \circ f)(x) = g(1-x^2) = 5$

$(f \circ f)(x) = f(1-x^2) = 1-(1-x^2)^2 = 1-(1-2x^2+x^4) = 2x^2 - x^4$

$(g \circ g)(x) = g(5) = 5$

(d) $f(x) = \sqrt{x}$, $g(x) = x^2 - 4$

$(f \circ g)(x) = f(x^2-4) = \sqrt{x^2-4}$ $\quad$ $(g \circ f)(x) = g(\sqrt{x}) = (\sqrt{x})^2 - 4 = x - 4$

$(f \circ f)(x) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$

$(g \circ g)(x) = g(x^2-4) = (x^2-4)^2 - 4 = x^4 - 8x^2 + 16 - 4 = x^4 - 8x^2 + 12$

(e) $f(x) = 3x - 5$, $g(x) = \frac{1}{x}$

$(f \circ g)(x) = f(\frac{1}{x}) = 3(\frac{1}{x}) - 5 = \frac{3}{x} - 5$ $\quad$ $(g \circ f)(x) = g(3x-5) = \frac{1}{3x-5}$

$(f \circ f)(x) = f(3x-5) = 3(3x-5) - 5 = 9x - 20$

$(g \circ g)(x) = g(\frac{1}{x}) = \frac{1}{\frac{1}{x}} = x$

(f) $f(x) = \frac{1}{1-x}$, $g(x) = \frac{x-2}{x+2}$

$$(f \circ g)(x) = f\left(\frac{x-2}{x+2}\right) = \frac{1}{1-\frac{x-2}{x+2}} = \frac{1}{\frac{x+2-x+2}{x+2}} = \frac{x+2}{4}$$

$$(g \circ f)(x) = g\left(\frac{1}{1-x}\right) = \frac{\frac{1}{1-x}-2}{\frac{1}{1-x}+2} = \frac{\frac{1-2+2x}{1-x}}{\frac{1+2-2x}{1-x}} = \frac{2x-1}{3-2x}$$

$$(f \circ f)(x) = f\left(\frac{1}{1-x}\right) = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{1-x-1} = \frac{x-1}{x}$$

$$(g \circ g)(x) = g\left(\frac{x-2}{x+2}\right) = \frac{\frac{x-2}{x+2}-2}{\frac{x-2}{x+2}+2} = \frac{\frac{x-2-2x-4}{x+2}}{\frac{x-2+2x+4}{x+2}} = \frac{-x-6}{3x+2}$$

(g) $f(x) = \sqrt{x}$, $g(x) = \sqrt{1+x}$

$(f \circ g)(x) = f\left(\sqrt{1+x}\right) = \sqrt{\sqrt{1+x}} = \sqrt[4]{1+x}$

$(g \circ f)(x) = g(\sqrt{x}) = \sqrt{1+\sqrt{x}}$

$(f \circ f)(x) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$ $\quad$ $(g \circ g)(x) = g\left(\sqrt{1+x}\right) = \sqrt{1+\sqrt{1+x}}$

2. (a) $h(x) = (2x+1)^9$, so $f(x) = x^9$, $g(x) = 2x+1$

(b) $h(x) = 1 + 2x^2 + 3x^4$, so $f(x) = 1 + 2x + 3x^2$, $g(x) = x^2$

(c) $h(x) = \frac{1}{x^2-7}$, so $f(x) = \frac{1}{x}$, $g(x) = x^2 - 7$

(d) $h(x) = \sqrt{6+x}$, so $f(x) = \sqrt{x}$, $g(x) = 6 + x$

Exercise 2.1

1. (a) $\lim_{h\to0}\frac{(3+h)^2-3^2}{h}$: $f(x)=x^2$, $a=3$ [Note : answers to this type of question are not unique, e.g. another possible answer is $g(x)=(3+x)^2$, $a=0$]

(b) $\lim_{h\to0}\frac{(2+h)^3-8}{h}$: $f(x)=x^3$, $a=2$

(c) $\lim_{h\to0}\frac{\sqrt{4+h}-2}{h}$: $f(x)=\sqrt{x}$, $a=4$

(d) $\lim_{h\to0}\frac{[(1+h)^4-3(1+h)]-4}{h}$: $f(x)=x^4+3x$, $a=1$

(e) $\lim_{h\to0}\frac{2^{1+h}-2}{h}$: $f(x)=2^x$, $a=1$

(f) $\lim_{x\to1}\frac{x^5-1}{x-1}$: $f(x)=x^5$, $a=1$

2. (a), (ii) (b), (i) (c), (iii)

3. See graphs : The functions are not differentiable where there are kinks or corners.

(a) $x=-3, 0, 2, 4$

(b) $x=-4, -2, 0, 2, 4, 6$

4. $f(x)=x^2+7x$, $f'(3)=\lim_{h\to0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to0}\frac{(3+h)^2+7(3+h)-[3^2+7(3)]}{h}$

$=\lim_{h\to0}\frac{h^2+6h+9+21+7h-30}{h}=\lim_{h\to0}\frac{h^2+13h}{h}=\lim_{h\to0}(h+13)=13$

5. $g(x)=15-3x^2$, $g'(-1)=\lim_{h\to0}\frac{g(-1+h)-g(-1)}{h}$

$=\lim_{h\to0}\frac{15-3(-1+h)^2-[15-3(-1)^2]}{h}$

$=\lim_{h\to0}\frac{15-3h^2+6h-3-12}{h}=\lim_{h\to0}\frac{6h-3h^2}{h}=\lim_{h\to0}(6-3h)=6$

6. $f(x)=\frac{1}{x}$, $(3,\frac{1}{3})$: $f'(3)=\lim_{h\to0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h}=\lim_{h\to0}\frac{\frac{3-3-h}{9+3h}}{h}$

$=\lim_{h\to0}-\frac{1}{9+3h}=-\frac{1}{9}$. At $(3,\frac{1}{3})$, $y-\frac{1}{3}=-\frac{1}{9}(x-3)\Rightarrow y=-\frac{1}{9}x+\frac{2}{3}$ or $x+9y-6=0$.

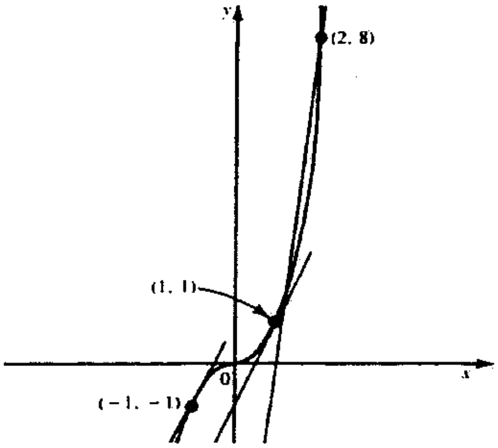
7. $f(x)=x^3$, $f'(a)=\lim_{h\to0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to0}\frac{(a+h)^3-a^3}{h}$

$=\lim_{h\to0}\frac{a^3+3a^2h+3ah^2+h^3-a^3}{h}$

$$= \lim_{h \to 0} \frac{3a^2h + 3ah^2 + h^3}{h} = \lim_{h \to 0} (3a^2 + 3ah + h^2) = 3a^2$$

At $(-1, -1)$, $m = 3(-1)^2 = 3$ At $(0,0)$, $m = 3(0)^2 = 0$

At $(1,1)$, $m = 3(1)^2 = 3$ At $(2,8)$, $m = 3(2)^2 = 12$



8. (a) $f(x) = 7x - x^2$, $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{7(a+h) - (a+h)^2 - [7a - a^2]}{h}$

$$= \lim_{h \to 0} \frac{7a + 7h - a^2 - 2ah - h^2 - 7a + a^2}{h} = \lim_{h \to 0} \frac{-2ah - h^2 + 7h}{h}$$

$$= \lim_{h \to 0} (-2a - h + 7) = 7 - 2a$$

(b) $f(x) = 2x^3 + 5$, $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{2(a+h)^3 + 5 - [2a^3 + 5]}{h}$

$$= \lim_{h \to 0} \frac{2a^3 + 6a^2h + 6ah^2 + h^3 + 5 - 2a^3 - 5}{h} = \lim_{h \to 0} \frac{6a^2h + 6ah^2 + h^3}{h}$$

$$= \lim_{h \to 0} (6a^2 + 6ah + h^2) = 6a^2$$

(c) $f(x) = \frac{1+2x}{1+x}$, $f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\frac{1+2(a+h)}{1+a+h} - \frac{1+2a}{1+a}}{h}$

$$= \lim_{h \to 0} \frac{\frac{(1+2a+2h)(1+a) - (1+2a)(1+a+h)}{(1+a+h)(1+a)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1+2a+2h+a+2a^2+2ah-1-a-h-2a-2a^2-2ah}{(1+a+h)(1+a)}}{h} = \lim_{h \to 0} \frac{\frac{h}{(1+a+h)(1+a)}}{h}$$

$$= \lim_{h \to 0} \frac{1}{(1+a+h)(1+a)} = \frac{1}{(1+a)^2}$$

(d) $f(x) = \sqrt{x}$, $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h}\times\frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}} = \lim_{h\to 0} \frac{a+h-a}{h\left(\sqrt{a+h}+\sqrt{a}\right)} = \lim_{h\to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

$$= \frac{1}{2\sqrt{a}}$$

9. $s = f(t) = 5t^2-2t+6,\ f'(a) = v(a) = \lim_{h\to 0} \frac{5(a+h)^2-2(a+h)+6-[5a^2-2a+6]}{h}$

$$= \lim_{h\to 0} \frac{5a^2+10ah+5h^2-2a-2h+6-5a^2+2a-6}{h} = \lim_{h\to 0} \frac{10ah+5h^2-2h}{h}$$

$$= \lim_{h\to 0} (10a+5h-2) = 10a-2$$

$v(1) = 10(1)-2 = 8$ m/s $\quad v(2) = 10(2)-2 = 18$ m/s $\quad v(3) = 10(3)-2 = 28$ m/s

10. (a) $f(x) = 3x^2+2x-4,\ f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{3(x+h)^2+2(x+h)-4-[3x^2+2x-4]}{h}$$

$$= \lim_{h\to 0} \frac{3x^2+6xh+3h^2+2x+2h-4-3x^2-2x+4}{h} = \lim_{h\to 0} \frac{6xh+3h^2+2h}{h}$$

$$= \lim_{h\to 0} (6x+3h+2) = 6x+2$$

(b) $f(x) = x^2-x^3,\ f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-(x+h)^3-[x^2-x^3]}{h}$

$$= \lim_{h\to 0} \frac{(x+h)^2-(x+h)^3-[x^2-x^3]}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^3-3x^2h-3xh^2-h^3-x^2+x^3}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2-3x^2h-3xh^2-h^3}{h} = \lim_{h\to 0} (2x+h-3x^2-3xh-h^2) = 2x-3x^2$$

(c) $f(x) = x^4,\ f'(x) = \lim_{h\to 0} \frac{(x+h)^4-x^4}{h} = \lim_{h\to 0} \frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}$

$$= \lim_{h\to 0} \frac{4x^3h+6x^2h^2+4xh^3+h^4}{h} = \lim_{h\to 0} (4x^3+6x^2h+4xh^2+h^3) = 4x^3$$

(d) $f(x) = \frac{x}{5x-1},\ f'(x) = \lim_{h\to 0} \frac{\frac{x+h}{5(x+h)-1}-\frac{x}{5x-1}}{h} = \lim_{h\to 0} \frac{\frac{(x+h)(5x-1)-x(5x+5h-1)}{(5x+5h-1)(5x-1)}}{h}$

$$= \lim_{h\to 0} \frac{\frac{5x^2-x+5hx-h-5x^2-5hx+x}{(5x+5h-1)(5x-1)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{(5x+5h-1)(5x-1)}}{h}$$

$$= \lim_{h\to 0} \frac{-1}{(5x+5h-1)(5x-1)} = \frac{-1}{(5x-1)^2}$$

Exercise 2.1

11. (a) $f(x) = \sqrt{2x-1}$, $f'(x) = \lim_{h \to 0} \frac{\sqrt{2(x+h)-1} - \sqrt{2x-1}}{h} \times \frac{\sqrt{2(x+h)-1} + \sqrt{2x-1}}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}$

$$= \lim_{h \to 0} \frac{2x+2h-1 - 2x+1}{h\left(\sqrt{2(x+h)-1} + \sqrt{2x-1}\right)} = \lim_{h \to 0} \frac{2h}{h\left(\sqrt{2(x+h)-1} + \sqrt{2x-1}\right)}$$

$$= \lim_{h \to 0} \frac{2}{\sqrt{2(x+h)-1} + \sqrt{2x-1}} = \frac{1}{\sqrt{2x-1}}$$

$\text{dom}(f) = \{x \mid 2x-1 \geq 0\} = \{x \mid x \geq \frac{1}{2}\}$, $\text{dom}(f') = \{x \mid x > \frac{1}{2}\}$.

(b) $g(x) = \frac{1}{\sqrt{x}}$, $g'(x) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \times \frac{\frac{1}{\sqrt{x+h}} + \frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x+h}} + \frac{1}{\sqrt{x}}}$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h\left(\frac{1}{\sqrt{x+h}} + \frac{1}{\sqrt{x}}\right)} = \lim_{h \to 0} \frac{\frac{x - x - h}{x(x+h)}}{h\left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x(x+h)}}\right)}$$

$$= \lim_{h \to 0} \frac{-1}{\left(\sqrt{x} + \sqrt{x+h}\right)\left(\sqrt{x^2+xh}\right)} = -\frac{1}{2\sqrt{x}\,x} = -\frac{1}{2\sqrt{x^3}}$$

$\text{dom}(g) = \text{dom}(g') = \{x \mid x > 0\}$

(c) $F(x) = \frac{3-2x}{4+x}$,

$$F'(x) = \lim_{h \to 0} \frac{\frac{3-2(x+h)}{4+x+h} - \frac{3-2x}{4+x}}{h} = \lim_{h \to 0} \frac{\frac{(3-2x-2h)(4+x)-(3-2x)(4+x+h)}{(4+x+h)(4+x)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{12+3x-8x-2x^2-8h-2hx-12-3x-3h+8x+2x^2+2hx}{(4+x+h)(4+x)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-11h}{(4+x+h)(4+x)}}{h} = \lim_{h \to 0} \frac{-11}{(4+x+h)(4+x)} = -\frac{11}{(4+x)^2}$$

$\text{dom}(F) = \text{dom}(F') = \{x \mid x \neq -4\}$

(d) $f(t) = \frac{2}{t^2-1}$, $f'(t) = \lim_{h \to 0} \frac{\frac{2}{(t+h)^2-1} - \frac{2}{t^2-1}}{h} = \lim_{h \to 0} \frac{\frac{2t^2-2-2t^2-4th-2h^2+2}{(t^2+2ht+h^2-1)(t^2-1)}}{h}$

$$= \lim_{h \to 0} \frac{\frac{-4th-2h^2}{(t^2+2ht+h^2-1)(t^2-1)}}{h} = \lim_{h \to 0} \frac{-4t-2h}{(t^2+2ht+h^2-1)(t^2-1)} = -\frac{4t}{(t^2-1)^2}$$

$\text{dom}(f) = \text{dom}(f') = \{t \mid t \neq \pm 1\}$

Exercise 2.1

12. (a) $y = 7 - 3x$, $\frac{dy}{dx} = \lim_{h \to 0} \frac{7 - 3(x+h) - [7 - 3x]}{h} = \lim_{h \to 0} \frac{7 - 3x - 3h - 7 + 3x}{h} = -3$

(b) $y = 3x^3 + 2x$, $\frac{dy}{dx} = \lim_{h \to 0} \frac{3(x+h)^3 + 2(x+h) - [3x^3 + 2x]}{h}$

$= \lim_{h \to 0} \frac{3x^3 + 9x^2h + 9xh^2 + 3h^3 + 2x + 2h - 3x^3 - 2x}{h} = \lim_{h \to 0} \frac{9x^2h + 9xh^2 + 3h^3 + 2h}{h}$

$= \lim_{h \to 0} (9x^2 + 9xh + 3h^2 + 2) = 9x^2 + 2$

(c) $y = x + \frac{1}{x}$, $\frac{dy}{dx} = \lim_{h \to 0} \frac{x + h + \frac{1}{x+h} - x - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{h + \frac{x - x - h}{x(x+h)}}{h}$

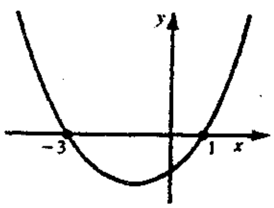
$= \lim_{h \to 0} \left(1 - \frac{1}{x^2 + xh}\right) = 1 - \frac{1}{x^2}$

(d) $y = \frac{1}{x^2}$, $\frac{dy}{dx} = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{\frac{1}{x^2 + 2xh + h^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{\frac{x^2 - x^2 - 2xh - h^2}{x^2(x^2 + 2xh + h^2)}}{h}$

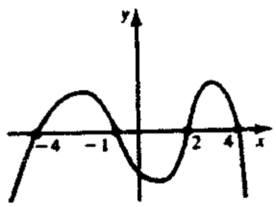
$= \lim_{h \to 0} \frac{-2x - h}{x^2(x^2 + 2xh + h^2)} = \frac{-2x}{x^4} = -\frac{2}{x^3}$

13. Using the given graphs as in Example 6, we get:

(a)

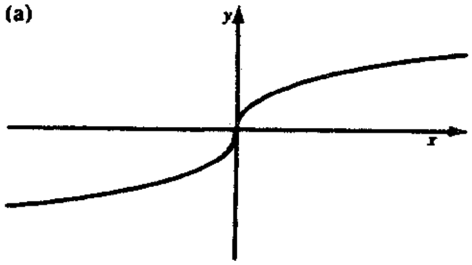


(b)



14. (a) $f(x) = \sqrt[3]{x}$

(a)



(b) $f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{\sqrt[3]{0+h} - \sqrt[3]{0}}{h} = \lim_{h \to 0} \frac{\sqrt[3]{h}}{h} = \lim_{h \to 0} \frac{1}{h^{\frac{2}{3}}}$.

This is undefined, so $f'(0)$ doesn't exist.

(c) If $a \neq 0$, $f'(a) = \lim\limits_{h \to 0} \dfrac{\sqrt[3]{a+h} - \sqrt[3]{a}}{h} \times \dfrac{\sqrt[3]{(a+h)^2} + \sqrt[3]{a^2+ah} + \sqrt[3]{a^2}}{\sqrt[3]{(a+h)^2} + \sqrt[3]{a^2+ah} + \sqrt[3]{a^2}}$

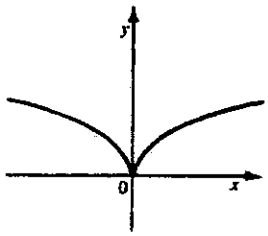
$$= \lim_{h \to 0} \frac{a+h-a}{h\left(\sqrt[3]{(a+h)^2} + \sqrt[3]{a^2+ah} + \sqrt[3]{a^2}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\left(\sqrt[3]{(a+h)^2} + \sqrt[3]{a^2+ah} + \sqrt[3]{a^2}\right)}$$

$$= \frac{1}{3\sqrt[3]{a^2}}$$

15. (a) $f(x) = x^{\frac{2}{3}}$, $f'(0) = \lim\limits_{h \to 0} \dfrac{(0+h)^{\frac{2}{3}} + (0)^{\frac{2}{3}}}{h} = \lim\limits_{h \to 0} \dfrac{h^{\frac{2}{3}}}{h} = \lim\limits_{h \to 0} \dfrac{1}{h^{\frac{1}{3}}}$, so $f'(0)$ does not exist.

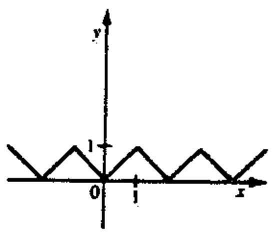
(b)



16. $f(x) = |x|$ if $-1 \leq x \leq 1$

$f(x+2) = f(x)$ for all values of x

(a)



(b) f is not differentiable for all $x \in I$

Exercise 2.2

1. (a) $f(x) = 32$, $f'(x) = 0$

(b) $f(x) = x^4$, $f'(x) = 4x^3$

(c) $y = x^{12}$, $y' = 12x^{11}$

(d) $y = -3.724$, $y' = 0$

(e) $f(x) = x$, $f'(x) = 1$

(f) $f(x) = x^{\pi}$, $f'(x) = \pi x^{\pi-1}$

(g) $f(x) = x^{43}$, $f'(x) = 43x^{42}$

(h) $f(x) = 2^5$, $f'(x) = 0$

(i) $g(x) = x^{-2}$, $g'(x) = -2x^{-3}$

(j) $g(x) = x^{\frac{3}{2}}$, $g'(x) = \frac{3}{2}x^{\frac{1}{2}}$

2. (a) $f(x) = 8x^{12}$, $f'(x) = 96x^{11}$

(b) $f(x) = -3x^9$, $f'(x) = -27x^8$

(c) $f(t) = 3t^{\frac{4}{3}}$, $f'(t) = 4t^{\frac{1}{3}}$

(d) $g(t) = 8t^{-\frac{3}{4}}$, $g'(t) = -6t^{-\frac{7}{4}}$

(e) $y = \frac{1}{x^4} = x^{-4}$, $y' = -4x^{-5} = -\frac{4}{x^5}$

(f) $y = \frac{2}{x^2} = 2x^{-2}$, $y' = -4x^{-3} = -\frac{4}{x^3}$

(g) $g(t) = (2t)^3 = 8t^3$, $g'(t) = 24t^2$

(h) $h(y) = (\frac{y}{3})^2 = \frac{y^2}{9}$, $h'(y) = \frac{2y}{9}$

(i) $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$, $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

(j) $f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$, $f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$

(k) $y = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$, $y' = -\frac{1}{2}x^{-\frac{3}{2}}$

(l) $y = \frac{3}{\sqrt[4]{x}} = 3x^{-\frac{1}{4}}$, $y' = -\frac{3}{4}x^{-\frac{5}{4}}$

(m) $y = \sqrt{3}x^{\sqrt{2}}$, $y' = \sqrt{6}\,x^{\sqrt{2}-1}$

(n) $y = (x^3)^4 = x^{12}$, $y' = 12x^{11}$

3. (a) $f(x) = 2x^3$, $x = \frac{1}{3}$; $f'(x) = 6x^2$, $m = f'(\frac{1}{3}) = 6(\frac{1}{3})^2 = \frac{2}{3}$

(b) $f(x) = x^{1.4}$, $x = 1$; $f'(x) = 1.4x^{0.4}$, $m = f'(1) = 1.4(1)^{0.4} = 1.4$

(c) $g(x) = x^{-3}$, $x = -1$; $g'(x) = -3x^{-4}$, $m = g'(-1) = -3(-1)^{-4} = -3$

(d) $g(x) = \sqrt[5]{x} = x^{\frac{1}{5}}$, $x = 32$; $g'(x) = \frac{1}{5}x^{-\frac{4}{5}}$, $m = g'(32) = \frac{1}{5}(32)^{-\frac{4}{5}} = \frac{1}{80}$

(e) $y = \sqrt{x^3} = x^{\frac{3}{2}}$, $x = 8$; $y' = \frac{3}{2}\sqrt{x}$, $m = y'(8) = \frac{3}{2}\sqrt{8} = 3\sqrt{2}$

(f) $y = \frac{6}{x} = 6x^{-1}$, $x = -3$; $y' = -6x^{-2}$, $m = y'(-3) = -6(-3)^{-2} = -\frac{2}{3}$

4. (a) $y = x^5$, $(2,32)$; $y' = 5x^4$, $m = y'(2) = 5(2)^4 = 80$, $y-32 = 80(x-2) \Rightarrow$

$y = 80x - 128$ or $80x - y - 128 = 0$.

(b) $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$, $(9,6)$; $y' = x^{-\frac{1}{2}}$, $m = y'(9) = 9^{-\frac{1}{2}} = \frac{1}{3}$. $y - 6 = \frac{1}{3}(x-9)$

$\Rightarrow y = \frac{1}{3}x + 3$ or $x - 3y + 9 = 0$.

(c) $xy = 1 \Rightarrow y = x^{-1}$, $(5, \frac{1}{5})$; $y' = -x^{-2}$, $m = y'(5) = -(5)^{-2} = -\frac{1}{25}$,

$y - \frac{1}{5} = -\frac{1}{25}(x-5) \Rightarrow y = -\frac{1}{25}x + \frac{2}{5}$ or $x + 25y - 10 = 0$.

(d) $y = \sqrt[3]{x} = x^{\frac{1}{3}}$, $(-8, -2)$; $y' = \frac{1}{3}x^{-\frac{2}{3}}$, $m = y'(-8) = \frac{1}{3}(-8)^{-\frac{2}{3}} = \frac{1}{12}$,

$y + 2 = \frac{1}{12}(x+8) \Rightarrow y = \frac{1}{12}x - \frac{4}{3}$ or $x - 12y - 16 = 0$.

5. Show if $f(x) = \frac{1}{x}$ then $f'(x) = -\frac{1}{x^2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\frac{x - x - h}{x(x+h)}}{h} = \lim_{h \to 0} \frac{\frac{-h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} -\frac{1}{x^2 + hx} = -\frac{1}{x^2}.$$ So if $f(x) = \frac{1}{x}$, $f'(x) = -\frac{1}{x^2}$

6. Show if $f(x) = \sqrt{x}$, then $f'(x) = \frac{1}{2\sqrt{x}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x + h - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h} - \sqrt{x}\right)} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} - \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

7. $y = 3x^2$, slope $= 24$. Slope of tangent is $\frac{dy}{dx} = 6x = 24$ when $x = 4$. At $x = 4$, $y = 3(4)^2 = 48$. So slope $= 24$ at $(4, 48)$ on $y = 3x^2$.

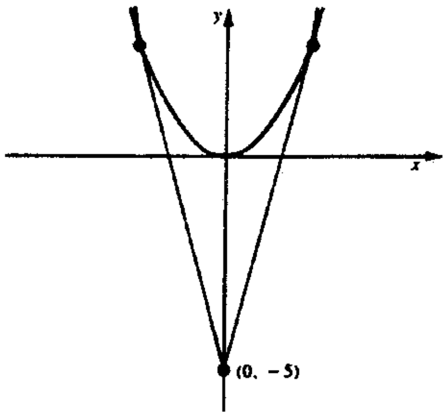
8. $y = x\sqrt{x} = x^{\frac{3}{2}}$, tangent $\parallel$ to $6x - y = 4$ or $y = 6x - 4$. So the tangent has a slope of 6. Slope of tangent is $\frac{dy}{dx} = \frac{3}{2}\sqrt{x} = 6$ when $x = 16$. At $x = 16$, $y = 16\sqrt{16} = 64$. So the tangent line is $\parallel$ to $6x - y = 4$ at $(16, 64)$ on $y = x\sqrt{x}$.

9. $y = -2x^4$, tangent $\perp$ to $x - y + 1 = 0 \Rightarrow y = x + 1$. The line has slope 1, so the tangent has slope -1 (negative reciprocal of 1). Slope of tangent is $\frac{dy}{dx} = -8x^3 = -1$ when $x = \frac{1}{2}$. At $x = \frac{1}{2}$, $y = -2(\frac{1}{2})^4 = -\frac{1}{8}$. So the tangent line is $\perp$ to $x - y + 1 = 0$ at $(\frac{1}{2}, -\frac{1}{8})$ on $y = -2x^4$.

Exercise 2.2

10. $y = 1 - \frac{1}{x}$, tangent $\perp$ to $y = 1 - 4x$. The line has slope -4, so the tangent has slope $\frac{1}{4}$ (negative reciprocal of -4). Slope of tangent is $\frac{dy}{dx} = \frac{1}{x^2} = \frac{1}{4}$ when $x = \pm 2$. At $x=2$, $y = 1 - \frac{1}{2} = \frac{1}{2}$. At $x = -2$, $y = 1 + \frac{1}{2} = \frac{3}{2}$. So the tangent lines are $\perp$ to $y = 1 - 4x$ at $(2, \frac{1}{2})$ and $(-2, \frac{3}{2})$ on $y = 1 - \frac{1}{x}$.

11. Let the x co-ord of the point be a. So the point is (a, a^2). The slope of the tangent at a is given by both $m = \frac{a^2+5}{a}$ (slope of line through (a,a^2) and $(0,-5)$) and $y' = 2x = 2a$ when $x = a$.
So $2a = \frac{a^2+5}{a} \Rightarrow a^2 = 5 \Rightarrow a = \pm\sqrt{5}$.
So the points are $(\sqrt{5}, 5)$, $(-\sqrt{5}, 5)$.
The slopes of the tangents are $\pm 2\sqrt{5}$, so the equations are $y - 5 = \pm 2\sqrt{5}(x \pm \sqrt{5})$ or $2\sqrt{5}x - y - 5 = 0$ and $2\sqrt{5}x + y + 5 = 0$.



12. $y = 8x^2$, $\tan\theta = 2.5$: Since $\tan\theta = \frac{OPP}{ADJ} = \frac{rise}{run}$, the slopes of the tangents are equal to $\tan\theta$, so the slopes are 2.5 and -2.5.
$\frac{dy}{dx} = 16x$, So $16x = 2.5 \Rightarrow x = \frac{5}{32}$, $16x = -2.5 \Rightarrow x = -\frac{5}{32}$. When $x = \frac{5}{32}$ (point Q), $y = 8(\frac{5}{32})^2 = \frac{50}{256}$. When $x = -\frac{5}{32}$ (point P), $y = 8(-\frac{5}{32})^2 = \frac{50}{256}$. So the points of contact are $P(-\frac{5}{32}, \frac{50}{256})$, $Q(\frac{5}{32}, \frac{50}{256})$.

Exercise 2.3

1. (a) $f(x) = x^2+4x,\ f'(x) = 2x+4$

(b) $f(x) = 3x^5-6x^4+2,\ f'(x) = 15x^4-24x^3$

(c) $g(x)=x^{10}+25x^5-50,\ g'(x)=10x^9+125x^4$

(d) $g(x) = x^2-\frac{2}{x^2} = x^2-2x^{-2},\ g'(x) = 2x+4x^{-3} = 2x+\frac{4}{x^3}$

(e) $h(x) = \sqrt{x} - 5x^4 = x^{\frac{1}{2}}-5x^4,\ h'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 20x^3 = \frac{1}{2\sqrt{x}} - 20x^3$

(f) $h(x) = (x-1)(x+6) = x^2+5x-6,\ h'(x) = 2x+5$

(g) $y = \frac{x+1}{\sqrt{x}} = x^{\frac{1}{2}}+x^{-\frac{1}{2}},\ y' = \frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}$

(h) $y = t^5-6t^{-5},\ y'= 5t^4+30t^{-6}$

(i) $f(t) = (1+t)^3 = t^3+3t^2+3t+1,\ f'(t) = 3t^2+6t+3$

(j) $F(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x} = x^{\frac{1}{2}}+x^{\frac{1}{3}}+x^{\frac{1}{4}},\ F'(x) = \frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{3}x^{-\frac{2}{3}}+\frac{1}{4}x^{-\frac{3}{4}}$

(k) $u(t) = a+\frac{b}{t}+\frac{c}{t^2} = a+bt^{-1}+ct^{-2},\ u'(t) = -bt^{-2}-2ct^{-3} = -\frac{b}{t^2}-\frac{2c}{t^3}$

(l) $v(r) = \sqrt{r}(2+3r) = 2r^{\frac{1}{2}} + 3r^{\frac{3}{2}},\ v'(r) = r^{-\frac{1}{2}} + \frac{9}{2}r^{\frac{1}{2}} = \frac{1}{\sqrt{r}} + \frac{9}{2}\sqrt{r}$

2. (a) $f(x) = 1+x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$, $D= R$; $f'(x) = 1+x+x^2+x^3$, $D= R$

(b) $f(x) = 4x- \sqrt[4]{x}$, $D= \{x \mid x\geq 0\}$; $f'(x) = 4-\frac{1}{4}x^{-\frac{3}{4}}$, $D= \{x \mid x>0\}$

(c) $f(x) = x+\frac{\sqrt{10}}{x^5}$, $D= \{x \mid x\neq 0\}$; $f'(x) = 1-\frac{5\sqrt{10}}{x^6}$, $D= \{x \mid x\neq 0\}$

(d) $f(x) = \sqrt{x} + \frac{2}{\sqrt{x}}$, $D= \{x \mid x>0\}$; $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - x^{-\frac{3}{2}}$, $D= \{x \mid x>0\}$

3. (a) $y = x^3-x^2+x-1,\ (1,0)$; $y' = 3x^2-2x+1$. When $x=1$,
$y'=3(1)^2-2(1)+1 = 2$. So $y = 2(x-1) \Rightarrow y = 2x-2$ or $2x-y-2 = 0$.

(b) $y = 7\sqrt{x} -3x,\ (1,4)$; $y'= \frac{7}{2}x^{-\frac{1}{2}} -3$. When $x=1$, $y'=\frac{7}{2}(1)^{-\frac{1}{2}}-3 = \frac{1}{2}$.
So $y-4 = \frac{1}{2}(x-1) \Rightarrow y = \frac{1}{2}x+\frac{7}{2}$ or $x-2y+7 = 0$.

(c) $y = x+\frac{6}{x}$, $(2,5)$; $y'=1-\frac{6}{x^2}$. When $x=2$, $y'= 1-\frac{6}{2^2} = -\frac{1}{2}$. So
$y-5 = -\frac{1}{2}(x-2) \Rightarrow y = -\frac{1}{2}x+6$ or $x+2y-12 = 0$.

(d) $y = (x^2+1)^2 = x^4+2x^2+1$, $(-1,4)$; $y' = 4x^3+4x$. When $x=-1$,
$y' = 4(-1)^3+4(-1) = -8$. So $y-4 = -8(x+1) \Rightarrow y = -8x-4$ or
$8x+y+4=0$.

4. $h = 40t-5t^2$; $v(t) = h'(t) = 40-10t$.
$v(2) = 40-10(2) = 20$ m/s, $v(4) = 40-10(4) = 0$ m/s, $v(5) = 40-10(5) = -10$ m/s

5. $s = 8t^2-5t+6$; $v(t) = s'(t) = 16t-5$
$v(1) = 16(1)-5 = 11$ m/s, $v(2) = 16(2)-5 = 27$ m/s, $v(5) = 16(5)-5 = 75$ m/s

6. $y = x^4-25x+2$, tangent ∥ to $7x-y=2 \Rightarrow y = 7x-2$. So the slope of the tangent is 7. Slope of tangent $y' = 4x^3-25 = 7 \Rightarrow 4x^3 = 32 \Rightarrow x=2$. At $x=2$, $y = 2^4-25(2)+2 = -32$. So the tangent is ∥ at $(2,-32)$.

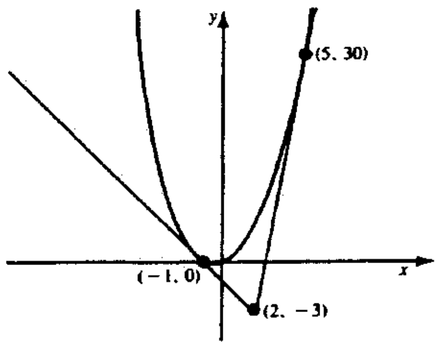
7. $y = x^3+3x^2-24x+1$, points with horizontal tangents $\Rightarrow y'=0$.
$y' = 3x^2+6x-24 = 0 \Rightarrow (x+4)(x-2) = 0 \Rightarrow x=-4,\ x=2$. At $x=-4$, $y=81$; $x=2$, $y=-27$. So the points where the tangents are horizontal are $(-4,81)$, $(2,-27)$.

8. Show $y = 10x^3+4x+2$ has no tangents with slope 3 $\Rightarrow y' \neq 3$.
$y' = 30x^2+4 = 3 \Rightarrow 30x^2=1 \Rightarrow x^2 = -\frac{1}{30}$, which is not possible. So the curve has no tangents with slope 3.

9. $y = 1+x^2$, tangents pass through origin.
Let x co-ord be a, so point on the curve is $(a,1+a^2)$. Slopes of the tangents are given by $m = \frac{1+a^2}{a}$ (slope of line between points $(a,1+a^2)$ and $(0,0)$) and by $y' = 2x = 2a$ when $x=a$. So $2a = \frac{1+a^2}{a} \Rightarrow a = \pm 1$. So the slopes of the tangents are ± 2. For $x=1$, $y-0 = 2(x-0) \Rightarrow y = 2x$, and for $x=-1$, $y-0 = -2(x-0) \Rightarrow y = -2x$.

10. $y = x^2+x$, tangents pass through $(2,-3)$. Let x co-ord be a, so tangent point on curve is (a,a^2+a). Slopes of tangents are given by $m = \frac{a^2+a+3}{a-2}$, and by $y' = 2x+1 = 2a+1$ when $x=a$. So $\frac{a^2+a+3}{a-2} = 2a+1 \Rightarrow a = -1, 5$.
For $a=-1$, $m=-1 \Rightarrow y+3 = -1(x-2) \Rightarrow y = -x-1$ or $x+y+1 = 0$.
For $a=5$, $m=11 \Rightarrow y+3 = 11(x-2) \Rightarrow y = 11x-25$ or $11x-y-25 = 0$.

Exercise 2.3



11. $xy = 1 \Rightarrow y = \frac{1}{x}$, tangents pass through $(1, -1)$, x co-ords only.

Let a be the x co-ord, so tangent point is $(a, \frac{1}{a})$. Slopes of tangents are given by

$m = \frac{\frac{1}{a}+1}{a-1}$ and $y' = -\frac{1}{x^2} = -\frac{1}{a^2}$ when $x = a$. So $\frac{\frac{1}{a}+1}{a-1} = -\frac{1}{a^2} \Rightarrow a + a^2 = -a + 1$

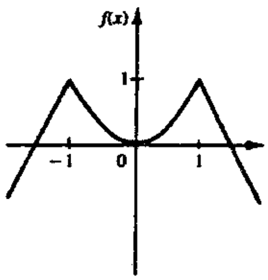
$\Rightarrow a^2 + 2a - 1 = 0$, so $a = -1 \pm \sqrt{2}$. So the x co-ords are $-1 \pm \sqrt{2}$.

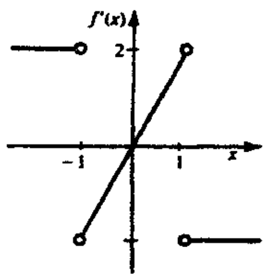
12. $f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 1 \\ 3-2x & \text{if } x > 1 \end{cases}$

(a) f is differentiable when $x < -1$, $-1 < x < 1$, and $x > 1$ since f is a polynomial in those intervals. We see that the graph of f has corners when $x = 1$ and -1, so f is not differentiable at $x = \pm 1$.

(b) $f(x) = 2x+3$ for $x < -1$, so $f'(x) = 2$ for $x < -1$. Similarly $f'(x) = 2x$ for $-1 < x < 1$ and $f'(x) = -2$ for $x > 1$.

Using this information, we have: $f'(x) = \begin{cases} 2 & \text{if } x < -1 \\ 2x & \text{if } -1 < x < 1 \\ -2 & \text{if } x > 1 \end{cases}$

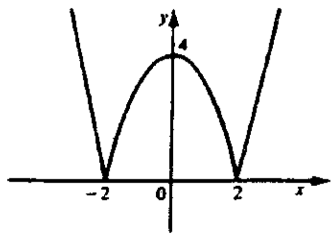




Exercise 2.3

13. $f(x) = |x^2 - 4|$

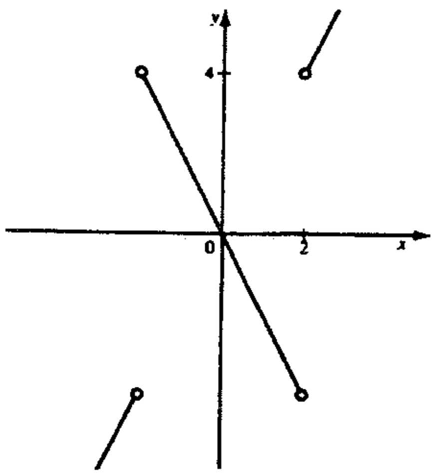
(a)



(b) From the graph, we see that $f'(x)$ will not exist at $x = \pm 2$ because the curve has a kink at these points.

(c) For $|x| \geq 2$, $f(x) = x^2 - 4$, so $f'(x) = 2x$ but only when $|x| > 2$ since $f'(2)$ and $f'(-2)$ don't exist. For $-2 < x < 2$, $f(x) = -x^2 + 4$, so $f'(x) = -2x$. This gives

$$f'(x) = \begin{cases} 2x & \text{if } x < -2 \\ -2x & \text{if } -2 < x < 2 \\ 2x & \text{if } x > 2 \end{cases}$$



Exercise 2.4

1. (a) $f(x) = (2x-1)(x^2+1)$, $f'(x) = (2x-1)\frac{d}{dx}(x^2+1)+(x^2+1)\frac{d}{dx}(2x-1)$
$= 2x(2x-1)+2(x^2+1)$

(b) $f(x) = x(3x-8)$, $f'(x) = x\frac{d}{dx}(3x-8)+(3x-8)\frac{d}{dx}x = 3x+(3x-8)$

(c) $y = x^2(1+x-3x^2)$, $y' = x^2\frac{d}{dx}(1+x-3x^2)+(1+x-3x^2)\frac{d}{dx}x^2$
$= x^2(1-6x)+2x(1+x-3x^2)$

(d) $y = (x^3+x^2+1)(x^2+2)$, $y' = (x^3+x^2+1)\frac{d}{dx}(x^2+2)+(x^2+2)\frac{d}{dx}(x^3+x^2+1)$
$= 2x(x^3+x^2+1)+(x^2+2)(3x^2+2x)$

(e) $f(t) = (t^4+t^2-1)(t^2-2)$, $f'(t) = (t^4+t^2-1)\frac{d}{dt}(t^2-2)+(t^2-2)\frac{d}{dt}(t^4+t^2-1)$
$= 2t(t^4+t^2-1)+(t^2-2)(4t^3+2t)$

(f) $f(t) = \sqrt[3]{t}(1-t)$, $f'(t) = \sqrt[3]{t}\frac{d}{dt}(1-t)+(1-t)\frac{d}{dt}\sqrt[3]{t} = -\sqrt[3]{t}+\frac{1}{3}t^{-\frac{2}{3}}(1-t)$

(g) $F(y) = \sqrt{y}(y-2\sqrt{y}+2)$, $F'(y) = \sqrt{y}\frac{d}{dy}(y-2\sqrt{y}+2)+(y-2\sqrt{y}+2)\frac{d}{dy}\sqrt{y}$
$= \sqrt{y}\left(1-\frac{1}{\sqrt{y}}\right)+\frac{1}{2\sqrt{y}}(y-2\sqrt{y}+2)$

(h) $G(y) = (y-y^2)(2y-y^{\frac{4}{3}})$, $G'(y) = (y-y^2)\frac{d}{dy}(2y-y^{\frac{4}{3}})+(2y-y^{\frac{4}{3}})\frac{d}{dy}(y-y^2)$
$= (y-y^2)(2-\frac{4}{3}y^{\frac{1}{3}})+(2y-y^{\frac{4}{3}})(1-2y)$

2. (a) $y = x^3(x^2+2x+3)$, $y' = x^3(2x+2)+3x^2(x^2+2x+3)$
$= 2x^4+2x^3+3x^4+6x^3+9x^2 = 5x^4+8x^3+9x^2$

(b) $y = x^{-2}(x^3-3x^2+6)$, $y' = x^{-2}(3x^2-6x)-2x^{-3}(x^3-3x^2+6)$
$= 3-6x^{-1}-2+6x^{-1}-12x^{-3} = 1-12x^{-3}$

(c) $f(x) = (1-x^2)(2-x^3)$,
$f'(x) = -3x^2(1-x^2)-2x(2-x^3) = -3x^2+3x^4-4x+2x^4 = 5x^4-3x^2-4x$

(d) $f(x) = (3x^3+4)(1-2x^3)$,
$f'(x) = -6x^2(3x^3+4)+9x^2(1-2x^3) = -18x^5-24x^2+9x^2-18x^5 = -36x^5-15x^2$

(e) $f(t) = (6+t^{-2})(8t^{10}-5t^3)$, $f'(t) = (6+t^{-2})(80t^9-15t^2)-2t^{-3}(8t^{10}-5t^3)$
$= 480t^9-90t^2+80t^7-15-16t^7+10 = 480t^9+64t^7-90t^2-5$

(f) $f(t) = (at+b)(ct^2-d)$, $f'(t) = 2ct(at+b)+a(ct^2-d) = 2act^2+2bct+act^2-ad$
$=3act^2+2bct-ad$

(g) $g(u) = \sqrt{u}(2-u^2+5u^4)$, $g'(u) = \sqrt{u}(-2u+20u^3)+\frac{1}{2}u^{-\frac{1}{2}}(2-u^2+5u^4)$
$= -2u^{\frac{3}{2}}+20u^{\frac{7}{2}}+u^{-\frac{1}{2}}-\frac{1}{2}u^{\frac{3}{2}}+\frac{5}{2}u^{\frac{7}{2}} = \frac{45}{2}u^{\frac{7}{2}}-\frac{5}{2}u^{\frac{3}{2}}+u^{-\frac{1}{2}}$

Exercise 2.4

(h) $g(v) = (v-\sqrt{v})(v^2+\sqrt{v})$, $g'(v) = (v-\sqrt{v})(2v+\frac{1}{2}v^{-\frac{1}{2}})+(1-\frac{1}{2}v^{-\frac{1}{2}})(v^2+\sqrt{v})$

$= 2v^2+\frac{1}{2}\sqrt{v} - 2v^{\frac{3}{2}} - \frac{1}{2} + v^2 - \frac{1}{2}v^{\frac{3}{2}} + \sqrt{v} - \frac{1}{2} = 3v^2 - \frac{5}{2}v^{\frac{3}{2}} + \frac{3}{2}\sqrt{v} - 1$

3. (a) $y = (1-2x)(3x-4)$, $x=2$; $y'=3(1-2x)-2(3x-4)$. At $x=2$,
$y'=3(-3)-2(2) = -13$
(b) $y = (1-x+x^2)(x-2)$, $x=1$; $y'=(1-x+x^2)+(2x-1)(x-2)$.
At $x=1$, $y'=(1)+(1)(-1) = 0$
(c) $y = x^4(4x^3+2)$, $x=-1$; $y'=x^4(12x^2)+4x^3(4x^3+2)$. At $x=-1$,
$y'=12-4(-2) = 20$
(d) $y = (1+x-2x^2)(3x^3+x-1)$, $x=1$;
$y'=(9x^2+1)(1+x-2x^2)+(1-4x)(3x^3+x-1)$.
At $x=1$, $y'=(10)(0)+(-3)(3) = -9$
(e) $y = x^{-5}(1+x^{-1})$, $x=1$; $y'=x^{-5}(-x^{-2})-5x^{-6}(1+x^{-1})$. At $x=1$,
$y'=-1-5(1+1) = -11$
(f) $y = (2-3\sqrt{x})(4-\sqrt{x})$, $x=4$; $y' = -\frac{1}{2}x^{-\frac{1}{2}}(2-3\sqrt{x})-\frac{3}{2}x^{-\frac{1}{2}}(4-\sqrt{x})$.
At $x=4$, $y' = -\frac{1}{2}\left(\frac{1}{2}\right)(-4)-\frac{3}{2}\left(\frac{1}{2}\right)(2) = -\frac{1}{2}$

4. $f(x) = (6x^4-3x^2+1)(2-x^3)$
(a) $f'(x) = -3x^2(6x^4-3x^2+1)+(2-x^3)(24x^3-6x)$, $f'(1) = -3(1)(4)+(1)(18) = 6$
(b) $f(x) = 12x^4-6x^7-6x^2+3x^5+2-x^3$, $f'(x) = -42x^6+15x^4+48x^3-3x^2-12x$,
$f'(1) = -42+15+48-3-12 = 6$

5. $y = (2-\sqrt{x})(1+\sqrt{x}+3x)$, $(1,5)$: $y'=(2-\sqrt{x})(\frac{1}{2}x^{-\frac{1}{2}}+3)-\frac{1}{2}x^{-\frac{1}{2}}(1+\sqrt{x}+3x)$.
At $x=1$, $y'=(1)(\frac{7}{2})-\frac{1}{2}(5) = 1$. So $y-5 = x-1 \Rightarrow y = x+4$ or $x-y+4 = 0$.

6. $f(2) = 3$, $f'(2) = 5$, $g(2) = -1$, $g'(2) = -4$, find $(fg)'(2)$
$(fg)'(2) = f(2)g'(2)+f'(2)g(2) = (3)(-4)+(5)(-1) = -17$

7. (a) $g(x)=xf(x)$; $g'(x) = xf'(x)+f(x)$
(b) $h(x) = \sqrt{x}f(x)$; $h'(x) = \sqrt{x}f'(x)+\frac{1}{2\sqrt{x}}f(x)$
(c) $F(x) = x^c f(x)$; $F'(x) = x^c f'(x)+cx^{c-1}f(x)$

8. (a) Show $\frac{d}{dx}[f(x)]^2 = 2f(x)f'(x)$

Let $g = f$, then $\frac{d}{dx}fg = fg' + f'g = ff' + f'f = 2ff'$. So $\frac{d}{dx}[f(x)]^2 = 2f(x)f'(x)$.

(b) $y = (2+5x-x^3)^2$; $y' = 2(2+5x-x^3)\frac{d}{dx}(2+5x-x^3) = 2(2+5x-x^3)(5-3x^2)$

9. (a) Show $(fgh)' = f'gh + fg'h + fgh'$

$(fgh)' = f'(gh) + f(gh)' = f'(gh) + f(gh' + g'h) = f'gh + fg'h + fgh'$

(b) $y = \sqrt{x}(3x+5)(6x^2-5x+1)$

$y' = \sqrt{x}(3x+5)\frac{d}{dx}(6x^2-5x+1) + \sqrt{x}(6x^2-5x+1)\frac{d}{dx}(3x+5) + (3x+5)(6x^2-5x+1)\frac{d}{dx}\sqrt{x}$

$= \sqrt{x}(3x+5)(12x-5) + 3\sqrt{x}(6x^2-5x+1) + \frac{1}{2\sqrt{x}}(3x+5)(6x^2-5x+1)$

10. (a) Using $f = g = h$, show $\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x)$

$(fgh)' = fgh' + fg'h + f'gh = fff' + ff'f + f'ff = f^2f' + f^2f' + f^2f' = 3f^2f'$

(b) $y = (1+x^3+x^6)^3$; $y' = 3(1+x^3+x^6)^2\frac{d}{dx}(1+x^3+x^6) = 3(3x^2+6x^5)(1+x^3+x^6)^2$

11. Prove $\frac{d}{dx}x^n = nx^{n-1}$

Proof: Let $n = 1$

L.S. $= \frac{d}{dx}x^1 = 1$ R.S. $= (1)x^{(1)-1} = (1)x^0 = 1$

So true for $n = 1$.

Assume true for $n = k$, so $\frac{d}{dx}x^k = kx^{k-1}$

Let $n = k+1$

L.S. $= \frac{d}{dx}x^{k+1}$ R.S. $= (k+1)x^{(k+1)-1}$

$= \frac{d}{dx}(x^k x)$ $= (k+1)x^k$

$= x^k\frac{d}{dx}(x) + x\frac{d}{dx}(x^k)$

$= x^k + x(kx^{k-1})$

$= x^k + kx^k$

$= (k+1)x^k =$ R.S.

So, by Mathematical Induction, $\frac{d}{dx}x^n = nx^{n-1}$ where n is any positive integer.

Exercise 2.5

1. (a) $f(x)=\frac{x-1}{x+1}$, $f'(x)=\frac{(x+1)\frac{d}{dx}(x-1)-(x-1)\frac{d}{dx}(x+1)}{(x+1)^2}=\frac{x+1-(x-1)}{(x+1)^2}$

$=\frac{2}{(x+1)^2}$

(b) $f(x)=\frac{2x-1}{x^2+1}$, $f'(x)=\frac{(x^2+1)\frac{d}{dx}(2x-1)-(2x-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}=\frac{2(x^2+1)-2x(2x-1)}{(x^2+1)^2}$

$=\frac{2x^2+2-4x^2+2x}{(x^2+1)^2}=\frac{-2x^2+2x+2}{(x^2+1)^2}$

(c) $g(x)=\frac{x}{x^2+2x-1}$, $g'(x)=\frac{(x^2+2x-1)\frac{d}{dx}(x)-x\frac{d}{dx}(x^2+2x-1)}{(x^2+2x-1)^2}$

$=\frac{x^2+2x-1-x(2x+2)}{(x^2+2x-1)^2}=\frac{x^2+2x-1-2x^2-2x}{(x^2+2x-1)^2}=\frac{-x^2-1}{(x^2+2x-1)^2}$

(d) $g(x)=\frac{x^3-1}{x^2+x+1}$, $g'(x)=\frac{(x^2+x+1)\frac{d}{dx}(x^3-1)-(x^3-1)\frac{d}{dx}(x^2+x+1)}{(x^2+x+1)^2}$

$=\frac{3x^2(x^2+x+1)-(x^3-1)(2x+1)}{(x^2+x+1)^2}=\frac{3x^4+3x^3+3x^2-(2x^4+x^3-2x-1)}{(x^2+x+1)^2}$

$=\frac{x^4+2x^3+3x^2+2x+1}{(x^2+x+1)^2}=\frac{(x^2+x+1)^2}{(x^2+x+1)^2}=1$

or simplify g(x) first, $g(x)=\frac{x^3-1}{x^2+x+1}=\frac{(x-1)(x^2+x+1)}{x^2+x+1}=x-1$, $g'(x)=1$.

(e) $y=\frac{\sqrt{x}}{x^2+1}$, $y'=\frac{(x^2+1)\frac{d}{dx}\sqrt{x}-\sqrt{x}\frac{d}{dx}(x^2+1)}{(x^2+1)^2}=\frac{\frac{1}{2\sqrt{x}}(x^2+1)-2x\sqrt{x}}{(x^2+1)^2}=\frac{x^2+1-4x^2}{2\sqrt{x}(x^2+1)^2}$

$=\frac{1-3x^2}{2\sqrt{x}(x^2+1)^2}$

(f) $y=\frac{\sqrt{x}+2}{\sqrt{x}-2}$, $y'=\frac{(\sqrt{x}-2)\frac{d}{dx}(\sqrt{x}+2)-(\sqrt{x}+2)\frac{d}{dx}(\sqrt{x}-2)}{(\sqrt{x}-2)^2}$

$=\frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-2)-\frac{1}{2\sqrt{x}}(\sqrt{x}+2)}{(\sqrt{x}-2)^2}=\frac{\sqrt{x}-2-(\sqrt{x}+2)}{2\sqrt{x}(\sqrt{x}-2)^2}=-\frac{2}{\sqrt{x}(\sqrt{x}-2)^2}$

(g) $f(t)=\frac{2t+1}{t^2-3t+4}$, $f'(t)=\frac{(t^2-3t+4)\frac{d}{dt}(2t+1)-(2t+1)\frac{d}{dt}(t^2-3t+4)}{(t^2-3t+4)^2}$

$=\frac{2(t^2-3t+4)-(2t+1)(2t-3)}{(t^2-3t+4)^2}=\frac{2t^2-6t+8-4t^2+4t+3}{(t^2-3t+4)^2}=\frac{-2t^2-2t+11}{(t^2-3t+4)^2}$

(h) $g(t) = \frac{2t^2+3t+1}{t-1}$, $g'(t) = \frac{(t-1)\frac{d}{dt}(2t^2+3t+1)-(2t^2+3t+1)\frac{d}{dt}(t-1)}{(t-1)^2}$

$= \frac{(t-1)(4t+3)-(2t^2+3t+1)}{(t-1)^2} = \frac{4t^2-t-3-2t^2-3t-1}{(t-1)^2} = \frac{2t^2-4t-4}{(t-1)^2}$

(i) $f(x) = \frac{1}{x^4-x^2+1}$, $f'(x) = \frac{(x^4-x^2+1)\frac{d}{dx}(1) - \frac{d}{dt}(x^4-x^2+1)}{(x^4-x^2+1)^2} = \frac{-4x^3+2x}{(x^4-2x^2+1)^2}$

(j) $f(x) = \frac{ax+b}{cx+d}$, $f'(x) = \frac{(cx+d)\frac{d}{dx}(ax+b) - (ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2} = \frac{a(cx+d)-c(ax+b)}{(cx+d)^2}$

$= \frac{acx+ad-acx-bc}{(cx+d)^2} = \frac{ad-bc}{(cx+d)^2}$

(k) $f(x) = \frac{x^6}{x^5-10}$, $f'(x) = \frac{(x^5-10)\frac{d}{dx}x^6 - x^6\frac{d}{dx}(x^5-10)}{(x^5-10)^2} = \frac{6x^5(x^5-10)-x^6(5x^4)}{(x^5-10)^2}$

$= \frac{6x^{10}-60x^5-5x^{10}}{(x^5-10)^2} = \frac{x^{10}-60x^5}{(x^5-10)^2}$

(l) $f(x) = \frac{1-\frac{1}{x}}{x+1}$, $f'(x) = \frac{(x+1)\frac{d}{dx}(1-\frac{1}{x}) - (1-\frac{1}{x})\frac{d}{dx}(x+1)}{(x+1)^2} = \frac{x^{-2}(x+1)-(1-\frac{1}{x})}{(x+1)^2}$

$= \frac{x^{-1}+x^{-2}-1+x^{-1}}{(x+1)^2} = \frac{x^{-2}+2x^{-1}-1}{(x+1)^2} = \frac{1+2x-x^2}{x^2(x+1)}$

2. (a) $f(x) = \frac{2+x}{1-2x}$, $D = \{x \mid 1-2x \neq 0\} = \{x \mid x \neq \frac{1}{2}\}$

$f'(x) = \frac{(1-2x)(1)+2(2+x)}{(1-2x)^2} = \frac{5}{(1-2x)^2}$

(b) $f(x) = \frac{x}{x^2-1}$, $D = \{x \mid x^2-1 \neq 0\} = \{x \mid x \neq \pm 1\}$,

$f'(x) = \frac{(x^2-1)-x(2x)}{(x^2-1)^2} = \frac{-x^2-1}{(x^2-1)^2}$

(c) $f(x) = \frac{1}{(x+1)(2x-3)}$, $D = \{x \mid x+1 \neq 0,\ 2x-3 \neq 0\} = \{x \mid x \neq -1,\ x \neq \frac{3}{2}\}$

$f'(x) = \frac{-[2(x+1)+(2x-3)]}{(x+1)^2(2x-3)^2} = \frac{-(2x+2+2x-3)}{(x+1)^2(2x-3)^2} = \frac{1-4x}{(x+1)^2(2x-3)^2}$

(d) $f(x) = \frac{2x+1}{x^2+2x-3}$,

$D = \{x \mid x^2+2x-3 \neq 0\} = \{x \mid (x+3)(x-1) \neq 0\} = \{x \mid x \neq -3,\ x \neq 1\}$

Exercise 2.5

$$f'(x) = \frac{2(x^2+2x-3)-(2x+1)(2x+2)}{(x^2+2x-3)^2} = \frac{2x^2+4x-6-4x^2-6x-2}{(x^2+2x-3)^2} = \frac{-2x^2-2x-8}{(x^2+2x-3)^2}$$

(e) $f(x) = \frac{x^2+2x}{x^4-1}$, $D = \{x \mid x^4-1 \neq 0\} = \{x \mid x \neq 1,\ x \neq -1\}$

$$f'(x) = \frac{(x^4-1)(2x+2)-4x^3(x^2+2x)}{(x^4-1)^2}$$

$$= \frac{2x^5+2x^4-2x-2-4x^5-8x^4}{(x^4-1)^2} = \frac{-2x^5-6x^4-2x-2}{(x^4-1)^2}$$

(f) $f(x) = \frac{x^2}{\sqrt{x}-3}$, $D = \{x \mid x \geq 0 \text{ and } \sqrt{x} \neq 3\} = \{x \mid x \geq 0,\ x \neq 9\}$

$$f'(x) = \frac{2x(\sqrt{x}-3)-x^2\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}-3)^2} = \frac{2x\sqrt{x}-6x-\frac{1}{2}x\sqrt{x}}{(\sqrt{x}-3)^2} = \frac{\frac{3}{2}x\sqrt{x}-6x}{(\sqrt{x}-3)^2}$$

$$= \frac{3x\sqrt{x}-12x}{2(\sqrt{x}-3)^2}$$

3. (a) $y = \frac{x}{x-2}$, $(4,2)$; $y' = \frac{x-2-x}{(x-2)^2} = -\frac{2}{(x-2)^2}$. At $x=4$, $y' = -\frac{2}{(4-2)^2}$

$= -\frac{1}{2}$. So $y-2 = -\frac{1}{2}(x-4) \Rightarrow y = -\frac{1}{2}x+4$ or $x+2y-8 = 0$.

(b) $y = \frac{1+3x}{2-3x}$, $(1,-4)$; $y' = \frac{3(2-3x)+3(1+3x)}{(2-3x)^2} = \frac{9}{(2-3x)^2}$. At $x=1$,

$y' = \frac{9}{(2-3)^2} = 9$. So $y+4 = 9(x-1) \Rightarrow y = 9x-13$ or $9x-y-13 = 0$.

(c) $y = \frac{1}{x^2+1}$, $(-2, \frac{1}{5})$; $y' = \frac{-2x}{(x^2+1)^2}$. At $x=-2$, $y' = \frac{-2(-2)}{(5)^2} = \frac{4}{25}$.

So $y-\frac{1}{5} = \frac{4}{25}(x+2) \Rightarrow y = \frac{4}{25}x+\frac{13}{25}$ or $4x-25y+13 = 0$.

(d) $y = \frac{x^3-1}{1+2x^2}$, $(1,0)$; $y' = \frac{3x^2(1+2x^2)-4x(x^3-1)}{(1+2x^2)^2} = \frac{3x^2+6x^4-4x^4+4x}{(1+2x^2)^2}$

$= \frac{2x^4+3x^2+4x}{(1+2x^2)^2}$. At $x=1$, $y' = \frac{2+3+4}{(3)^2} = 1$. So $y = x-1$ or $x-y-1 = 0$.

4. $f(2) = 3$, $f'(2) = 5$, $g(2) = -1$, $g'(2) = -4$

$$\left(\frac{f}{g}\right)'(2) = \frac{f'(2)g(2)-g'(2)f(2)}{[g(2)]^2} = \frac{(5)(-1)-(-4)(3)}{(-1)^2} = 7$$

5. $y = \frac{x+2}{3x+4}$; $y' = \frac{(3x+4)-3(x+2)}{(3x+4)^2} = -\frac{2}{(3x+4)^2}$. Since y' is always negative, y has no tangents with positive slopes.

6. $y = \frac{x^2}{2x+5}$, where are horizontal tangents? $y' = \frac{2x(2x+5)-2x^2}{(2x+5)^2} = \frac{2x^2+10x}{(2x+5)^2}$.
Horizontal tangent $\Rightarrow y' = 0 \Rightarrow 2x^2+10x = 0 \Rightarrow 2x(x+5) = 0 \Rightarrow x = 0, x = -5$.
When $x = 0$, $y = 0$; $x = -5$, $y = 5$. So the horizontal tangents occur at $(0,0)$, $(-5,5)$.

7. $y = \frac{x}{x-1}$, tangent parallel to $x+4y = 1 \Rightarrow y = -\frac{1}{4}x+\frac{1}{4}$. Slope of line is $-\frac{1}{4}$ so slope of tangent will be $-\frac{1}{4}$. Slope of tangent $y' = \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2} = -\frac{1}{4}$ when $(x-1)^2 = 4 \Rightarrow x = 3, x = -1$. When $x = 3$, $y = \frac{3}{2}$; $x = -1$, $y = \frac{1}{2}$.
So the tangent line is parallel to $x+4y = 1$ at $(3,\frac{3}{2})$, $(-1,\frac{1}{2})$.

8. (a) $y = \frac{1}{f(x)}$; $y' = \frac{(0)f(x)-f'(x)}{[f(x)]^2} = -\frac{f'(x)}{[f(x)]^2}$

(b) $y = \frac{f(x)}{x}$; $y' = \frac{xf'(x)-f(x)}{x^2}$

c) $y = \frac{x}{f(x)}$; $y' = \frac{f(x)-xf'(x)}{[f(x)]^2}$

9. Prove $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$

$$\frac{d}{dx}(x^{-n}) = \frac{d}{dx}\left(\frac{1}{x^n}\right)$$
$$= \frac{x^n\frac{d}{dx}(1)-\frac{d}{dx}(x^n)}{(x^n)^2}$$
$$= \frac{-nx^{n-1}}{x^{2n}}$$
$$= -nx^{n-1-2n}$$
$$= -nx^{-n-1}$$

Exercise 2.6

1. (a) $F(x) = (5-3x)^7$, $F'(x) = 7(5-3x)^6\frac{d}{dx}(5-3x) = -21(5-3x)^6$

(b) $F(x) = (2x^2+1)^{20}$, $F'(x) = 20(2x^2+1)^{19}\frac{d}{dx}(2x^2+1) = 80x(2x^2+1)^{19}$

(c) $G(x) = (x^3+x^2-2)^{\frac{3}{4}}$, $G'(x) = \frac{3}{4}(x^3+x^2-2)^{-\frac{1}{4}}\ \frac{d}{dx}(x^3+x^2-2) = \dfrac{9x^2+6x}{4\ \sqrt[4]{x^3+x^2-2}}$

(d) $G(x) = \sqrt{x^4-x+1} = (x^4-x+1)^{\frac{1}{2}}$, $G'(x) = \frac{1}{2}(x^4-x+1)^{-\frac{1}{2}}\frac{d}{dx}(x^4-x+1)$

$= \dfrac{4x^3-1}{2\sqrt{x^4-x+1}}$

(e) $y = \sqrt[4]{x^2+x} = (x^2+x)^{\frac{1}{4}}$, $y' = \frac{1}{4}(x^2+x)^{-\frac{3}{4}}\frac{d}{dx}(x^2+x) = \dfrac{2x+1}{4(x^2+x)^{\frac{3}{4}}}$

(f) $y = (1+3x+4x^2)^{-3}$, $y' = -3(1+3x+4x^2)^{-4}\frac{d}{dx}(1+3x+4x^2) = -\dfrac{3(3+8x)}{(1+3x+4x^2)^4}$

(g) $y = \dfrac{1}{(x^3+2x^2+1)^2} = (x^3+2x^2+1)^{-2}$, $y' = -2(x^3+2x^2+1)^{-3}\frac{d}{dx}(x^3+2x^2+1)$

$= -\dfrac{2(3x^2+4x)}{(x^3+2x^2+1)^3}$

(h) $y = \dfrac{4}{\sqrt{9-x^2}} = 4(9-x^2)^{-\frac{1}{2}}$, $y' = 4(-\frac{1}{2})(9-x^2)^{-\frac{3}{2}}\frac{d}{dx}(9-x^2) = \dfrac{4x}{(9-x^2)^{\frac{3}{2}}}$

(i) $y = (1+2\sqrt{x})^6$, $y' = 6(1+2\sqrt{x})^5\frac{d}{dx}(1+2\sqrt{x}) = \dfrac{6(1+2\sqrt{x})^5}{\sqrt{x}}$

(j) $y = \sqrt{x+\sqrt{x}} = (x+\sqrt{x})^{\frac{1}{2}}$, $y' = \frac{1}{2}(x+\sqrt{x})^{-\frac{1}{2}}\frac{d}{dx}(x+\sqrt{x}) = \dfrac{1}{2\sqrt{x+\sqrt{x}}}\times\left(1+\dfrac{1}{2\sqrt{x}}\right)$

$= \dfrac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$

(k) $y = x - \sqrt[5]{1+x^5-6x^{10}} = x-(1+x^5-6x^{10})^{\frac{1}{5}}$, $y' =$

$1-\frac{1}{5}(1+x^5-6x^{10})^{-\frac{4}{5}}\frac{d}{dx}(1+x^5-6x^{10})$

$= 1 - \dfrac{x^4-12x^9}{(1+x^5-6x^{10})^{\frac{4}{5}}}$

(l) $y = x^2+(x^2-1)^5$, $y' = 2x+5(x^2-1)^4\frac{d}{dx}(x^2-1) = 2x+10x(x^2-1)^4$

2. $y = u^4+5u^2$, $u = x^5+2x^2+1$; $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (4u^3+10u)(5x^4+4x)$

Exercise 2.6

3. $y = u^2 - 2u^5$, $u = x - \sqrt{x}$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (2u - 10u^4)(1 - \frac{1}{2\sqrt{x}})$. When $x = 4$,

$u = 4 - \sqrt{4} = 2$. So $\left[\frac{dy}{dx}\right]_{x=4} = [2(2) - 10(2)^4][1 - \frac{1}{2\sqrt{4}}] = (-156)(\frac{3}{4}) = -117.$

4. $y = \sqrt{1 + r^2}$, $r = \frac{t+1}{2t+1}$: $\frac{dy}{dt} = \frac{dy}{dr}\frac{dr}{dt} = \left(\frac{r}{\sqrt{1+r^2}}\right)\left(\frac{2t+1-2(t+1)}{(2t+1)^2}\right)$

$= \left(\frac{r}{\sqrt{1+r^2}}\right)\left(\frac{-1}{(2t+1)^2}\right)$. When $t = 1$, $r = \frac{1+1}{2+1} = \frac{2}{3}$.

So $\left[\frac{dy}{dt}\right]_{t=1} = \left(\frac{\frac{2}{3}}{\sqrt{1 + (\frac{2}{3})^2}}\right)\left(\frac{-1}{(2+1)^2}\right) = \frac{-\frac{2}{3}}{9\sqrt{\frac{13}{9}}} = -\frac{2}{9\sqrt{13}}.$

5. $s = v + \frac{50}{v}$, $v = 3t - \sqrt{t}$: $\frac{ds}{dt} = \frac{ds}{dv}\frac{dv}{dt} = \left(1 - \frac{50}{v^2}\right)\left(3 - \frac{1}{2\sqrt{t}}\right)$. When $t = 4$,

$v = 3(4) - \sqrt{4} = 12 - 2 = 10$. So $\left[\frac{ds}{dt}\right]_{t=4} = \left(1 - \frac{50}{10^2}\right)\left(3 - \frac{1}{2\sqrt{4}}\right) = (\frac{1}{2})(\frac{11}{4}) = \frac{11}{8}$.

6. (a) $F(x) = x\sqrt{x^2+1}$, $F'(x) = x\frac{2x}{2\sqrt{x^2+1}} + \sqrt{x^2+1} = \frac{2x^2+1}{\sqrt{x^2+1}}$

(b) $F(x) = (2x+1)(4x-1)^5$, $F'(x) = (2x+1)(5)(4)(4x-1)^4 + 2(4x-1)^5$

$= 2(20x + 10 + 4x - 1)(4x-1)^4 = 6(8x+3)(4x-1)^4$.

(c) $G(x) = (x^2-1)^4(2-3x)$, $G'(x) = -3(x^2-1)^4 + (2-3x)(4)(x^2-1)^3(2x)$

$= (-3x^2 + 3 + 16x - 24x^2)(x^2-1)^3 = (3 + 16x - 27x^2)(x^2-1)^3$

(d) $G(x) = (x^4 - x + 1)^2(x^2-2)^3$,

$G'(x) = (x^4 - x + 1)^2(3)(x^2-2)^2(2x) + 2(x^4 - x + 1)(4x^3 - 1)(x^2-2)^3$

$= 2(x^4 - x + 1)(x^2-2)^2[3x(x^4 - x + 1) + (x^2-2)(4x^3-1)]$

$= 2(x^4 - x + 1)(x^2-2)^2(7x^5 - 8x^3 - 4x^2 + 3x + 2)$

(e) $F(x) = \frac{x}{\sqrt{2x+3}}$, $F'(x) = \frac{\sqrt{2x+3} - x(\frac{1}{2})(2x+3)^{-\frac{1}{2}}(2)}{2x+3} = \frac{2x+3-x}{(2x+3)^{\frac{3}{2}}} = \frac{x+3}{(2x+3)^{\frac{3}{2}}}$

(f) $f(t) = \frac{(1+2t)^5}{(3t^2-5)^2}$, $f'(t) = \frac{5(1+2t)^4(2)(3t^2-5)^2 - 2(3t^2-5)(6t)(1+2t)^5}{(3t^2-5)^4}$

$= \frac{2(1+2t)^4[5(3t^2-5) - 6t(1+2t)]}{(3t^2-5)^3} = \frac{2(1+2t)^4(15t^2 - 25 - 6t - 12t^2)}{(3t^2-5)^3}$

$$= \frac{2(1+2t)^4(3t^2-6t-25)}{(3t^2-5)^3}$$

(g) $g(x) = \left(\frac{x+2}{x-2}\right)^3$, $g'(x) = 3\left(\frac{x+2}{x-2}\right)^2\left(\frac{x-2-(x+2)}{(x-2)^2}\right) = \frac{-12(x+2)^2}{(x-2)^4}$

(h) $h(t) = \left(\frac{t^2+1}{t+1}\right)^{10}$, $h'(t) = 10\left(\frac{t^2+1}{t+1}\right)^9\left(\frac{2t(t+1)-(t^2+1)}{(t+1)^2}\right) = 10\left(\frac{t^2+1}{t+1}\right)^9\left(\frac{t^2+2t-1}{(t+1)^2}\right)$

$$= \frac{10(t^2+1)^9(t^2+2t-1)}{(t+1)^{11}}$$

(i) $y = \sqrt{\frac{x^2-1}{x^2+1}}$, $y' = \frac{1}{2}\left(\frac{x^2-1}{x^2+1}\right)^{-\frac{1}{2}}\left(\frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2}\right) = \frac{1}{2}\left(\frac{x^2+1}{x^2-1}\right)^{\frac{1}{2}}\left(\frac{4x}{(x^2+1)^2}\right)$

$$= \frac{2x}{(x^2+1)^{\frac{3}{2}}\sqrt{x^2-1}}$$

(j) $y = \frac{(2x+3)^3}{\sqrt{4x-7}}$, $y' = \frac{3(2x+3)^2(2)\sqrt{4x-7} - \frac{1}{2}(4x-7)^{-\frac{1}{2}}(4)(2x+3)^3}{4x-7}$

$$= \frac{2(2x+3)^2[3(4x-7)-(2x+3)]}{(4x-7)^{\frac{3}{2}}} = \frac{4(2x+3)^2(5x-12)}{(4x-7)^{\frac{3}{2}}}$$

(k) $y = 3\sqrt{x}(2x+1)^5+\sqrt{4x-3}$, $y' = 3\sqrt{x}(5)(2x+1)^4(2)+\frac{3}{2}x^{-\frac{1}{2}}(2x+1)^5+\frac{1}{2}(4x-3)^{-\frac{1}{2}}(4)$

$$= \frac{3}{2\sqrt{x}}(2x+1)^4[20x+2x+1] + \frac{2}{\sqrt{4x-3}} = \frac{3(2x+1)^4[22x+1]}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

(l) $y = \sqrt{1+\sqrt[3]{x}}$, $y = \frac{1}{2}(1+\sqrt[3]{x})^{-\frac{1}{2}}(\frac{1}{3}x^{-\frac{2}{3}}) = \frac{1}{6x^{\frac{2}{3}}\sqrt{1+\sqrt[3]{x}}}$

(m) $y = (t+\sqrt[3]{t+t^2})^{20}$, $y' = 20\left(t+\sqrt[3]{t+t^2}\right)^{19}\left(1+\frac{1}{3}(t+t^2)^{-\frac{2}{3}}(1+2t)\right)$

$$= 20\left(t+\sqrt[3]{t+t^2}\right)^{19}\left(1+\frac{1+2t}{3(t+t^2)^{\frac{2}{3}}}\right)$$

(n) $y = \sqrt{x+\sqrt{x+\sqrt{x}}}$, $y' = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}(1+\frac{1}{2}(x+x^{\frac{1}{2}})^{-\frac{1}{2}}(1+\frac{1}{2}x^{-\frac{1}{2}}))$

$$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}\right)$$

Exercise 2.6

7. $y = (x^2-3)^8$, $(2,1)$; $y' = 8(x^2-3)^7(2x) = 16x(x^2-3)^7$. When $x=2$, $y' = 16(2)(4-3)^7 = 32$. So $y-1 = 32(x-2) \Rightarrow y = 32x-63$ or $32x-y-63 = 0$.

8. $y = \dfrac{1}{\sqrt{20-x^4}}$, $(2,\frac{1}{2})$; $y' = -\frac{1}{2}(20-x^4)^{-\frac{3}{2}}(-4x^3) = \dfrac{2x^3}{(20-x^4)^{\frac{3}{2}}}$.

When $x=2$, $y' = \dfrac{2(2)^3}{(20-2^4)^{\frac{3}{2}}} = \dfrac{16}{\sqrt{4^3}} = 2$.

So $y-\frac{1}{2} = 2(x-2) \Rightarrow y = 2x-\frac{7}{2}$ or $4x-2y-7 = 0$.

9. $F(x) = f(g(x))$, $g(2) = 4$, $g'(2) = 3$, $f'(4) = 5$.
$F'(x) = f'(g(x))g'(x)$; $F'(2) = f'(g(2))g'(2) = f'(4)(3) = (5)(3) = 15$

10. $G(x) = h(p(x))$, $h(5) = 1$, $h'(5) = 2$, $h'(1) = 3$, $p(1) = 5$, $p'(1) = 7$
$G'(x) = h'(p(x))p'(x)$; $G'(1) = h'(p(1))p'(1) = h'(5)(7) = (2)(7) = 14$

11. (a) $F(x) = f(x^4)$; $F'(x) = 4x^3f'(x^4)$ (b) $G(x) = [f(x)]^4$; $G'(x) = 4[f(x)]^3f'(x)$

(c) $H(x) = f(\sqrt{x})$; $H'(x) = \dfrac{1}{2\sqrt{x}}f'(\sqrt{x})$ (d) $P(x) = \sqrt{f(x)}$; $P'(x) = \dfrac{f'(x)}{2\sqrt{f(x)}}$

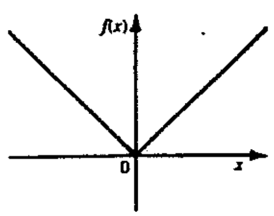
(e) $y = f(f(x))$; $y' = f'(f(x))f'(x)$ (f) $y = \sqrt{1+[f(x)]^2}$; $y' = \dfrac{f(x)f'(x)}{\sqrt{1+[f(x)]^2}}$

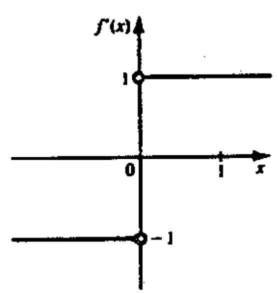
(g) $y = [f(x^2)]^2$; $y' = 2[f(x^2)]f'(x^2)(2x) = 4xf(x^2)f'(x^2)$

(h) $y = f([f(x)]^3)$; $y' = f'([f(x)]^3)(3)[f(x)]^2f'(x) = 3f'(x)[f(x)]^2f'([f(x)]^3)$

12. (a) Show $\frac{d}{dx}|x| = \frac{x}{|x|}$: $\frac{d}{dx}|x| = \frac{d}{dx}\sqrt{x^2} = \dfrac{1}{2\sqrt{x^2}}(2x) = \dfrac{x}{\sqrt{x^2}} = \dfrac{x}{|x|}$

(b)





(c) $g(x) = x|x|$; $g'(x) = x(\frac{x}{|x|}) + |x| = \dfrac{x^2+|x|^2}{|x|} = \dfrac{2|x|^2}{|x|} = 2|x|$.

Exercise 2.7

1. (a) $x^2 - y^2 = 1$

$$2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x}{y}$$

(b) $x^3 + y^3 = 6$

$$3x^2 + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x^2}{y^2}$$

(c) $xy = 4$

$$x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

(d) $x^2 + xy + y^2 = 1,$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 2y) = -y - 2x$$

$$\frac{dy}{dx} = -\frac{y + 2x}{x + 2y}$$

(e) $x^3 + y^3 = 6xy$

$$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$$

$$\frac{dy}{dx}(3y^2 - 6x) = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$$

(f) $2xy^2 - y^3 = x^2,$

$$2x(2y)\frac{dy}{dx} + 2y^2 - 3y^2\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(4xy - 3y^2) = 2x - 2y^2$$

$$\frac{dy}{dx} = \frac{2x - 2y^2}{4xy - 3y^2}$$

(g) $\sqrt{x} + \sqrt{y} = 1$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

(h) $\frac{2x}{x + y} = y$

$$\frac{2(x + y) - 2x(1 + \frac{dy}{dx})}{(x + y)^2} = \frac{dy}{dx}$$

$$\frac{2x - 2y - 2x - 2x\frac{dy}{dx}}{(x + y)^2} = \frac{dy}{dx}$$

$$\frac{2y}{(x + y)^2} = \frac{dy}{dx} + \frac{2x\frac{dy}{dx}}{(x + y)^2}$$

$$\frac{dy}{dx}\left[1 + \frac{2x}{(x + y)^2}\right] = \frac{2y}{(x + y)^2}$$

$$\frac{dy}{dx} = \frac{2y}{(x + y)^2 + 2x}$$

2.(a) $x^2 + 4y^2 = 5,\ (1, -1)$

$$2x + 8y\frac{dy}{dx} = 0,$$

$$\frac{dy}{dx} = -\frac{x}{4y}\ ;\ \text{at } (1, -1),\ \frac{dy}{dx} = \frac{1}{4}$$

(b) $x^4 + y^4 = 17,\ (2, 1)$

$$4x^3 + 4y^3\frac{dy}{dx} = 0,$$

$$\frac{dy}{dx} = -\frac{x^3}{y^3}\ ;\ \text{at } (2, 1),\ \frac{dy}{dx} = -8$$

(c) $x^2 + x^3y^2 - y^3 = 13,\ (1, -2)$

$$2x + x^3(2y)\frac{dy}{dx} + 3x^2y^2 - 3y^2\frac{dy}{dx} = 0,$$

$$\frac{dy}{dx}(2x^3y - 3y^2) = -3x^2y^2 - 2x,$$

$$\frac{dy}{dx} = -\frac{3x^2y^2 + 2x}{2x^3y - 3y^2}\ ,\ \text{at } (1, -2),$$

$$\frac{dy}{dx} = -\frac{3(1)^2(-2)^2 + 2(1)}{2(1)^2(-2) - 3(-2)^2} = \frac{7}{8}$$

(d) $y^2 = 2xy - 3,\ (2, 3)$

$$2y\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y,$$

$$\frac{dy}{dx}(2y - 2x) = 2y,$$

$$\frac{dy}{dx} = \frac{y}{y - x}\ ;\ \text{at } (2, 3),\ \frac{dy}{dx} = 3$$

(e) $\sqrt{x+y}+\sqrt{xy}=4,\ (2,2)$

$$\frac{1}{2\sqrt{x+y}}\left(1+\frac{dy}{dx}\right)+\frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx}+y\right)=0,$$

$$\frac{dy}{dx}\left[\frac{1}{2\sqrt{x+y}}+\frac{x}{2\sqrt{xy}}\right]=-\frac{1}{2\sqrt{x+y}}-\frac{y}{2\sqrt{xy}}$$

$$\frac{dy}{dx}\left[\frac{2\sqrt{xy}+2x\sqrt{x+y}}{4\sqrt{x+y}\sqrt{xy}}\right]=-\frac{2\sqrt{xy}+2y\sqrt{x+y}}{4\sqrt{x+y}\sqrt{xy}}$$

$$\frac{dy}{dx}=-\frac{2\sqrt{xy}+2y\sqrt{x+y}}{2\sqrt{xy}+2x\sqrt{x+y}}\ ;\ \text{at } (2,2),$$

$$\frac{dy}{dx}=-\frac{2\sqrt{(2)(2)}+2(2)\sqrt{2+2}}{2\sqrt{(2)(2)}+2(2)\sqrt{2+2}}=-1$$

(f) $\frac{1}{x}+\frac{1}{y}=1,\ (\frac{3}{2},3)$

$$-\frac{1}{x^2}-\frac{1}{y^2}\frac{dy}{dx}=0,$$

$$\frac{dy}{dx}=-\frac{y^2}{x^2}\ ;\ \text{at } (\tfrac{3}{2},3),\ \frac{dy}{dx}=-4$$

3. (a) $2x^2-y^2=1,\ (-1,-1)$

$$4x-2y\frac{dy}{dx}=0,$$

$$\frac{dy}{dx}=\frac{2x}{y}\ ;\ \text{at } (-1,-1),\ \frac{dy}{dx}=2$$

$y+1=2(x+1)\Rightarrow y=2x+1$

or $2x-y+1=0.$

(b) $x^3+y^3=9,\ (2,1)$

$$3x^2+3y^2\frac{dy}{dx}=0,$$

$$\frac{dy}{dx}=-\frac{x^2}{y^2}\ ;\ \text{at } (2,1),\ \frac{dy}{dx}=-4$$

$y-1=-4(x-2)\Rightarrow y=-4x+9$

or $4x+y-9=0.$

(c) $y^5+x^2y^3=10,\ (-3,1)$

$$5y^4\frac{dy}{dx}+3x^2y^2\frac{dy}{dx}+2xy^3=0$$

$$\frac{dy}{dx}(5y^4+3x^2y^2)=-2xy^3$$

$$\frac{dy}{dx}=-\frac{2xy}{5y^2+3x^2}\ ;\ \text{at } (-3,1),\ \frac{dy}{dx}=\frac{3}{16}$$

$y-1=\frac{3}{16}(x+3)\Rightarrow y=\frac{3}{16}x+\frac{25}{16}$

or $3x-16y+25=0.$

(d) $(x+y)^3=x^3+y^3\ ,\ (-1,1)$

$$3(x+y)^2\left(1+\frac{dy}{dx}\right)=3x^2+3y^2\frac{dy}{dx}$$

$$\frac{dy}{dx}[3(x+y)^2-3y^2]=3x^2-3(x+y)^2$$

$$\frac{dy}{dx}=\frac{x^2-(x+y)^2}{(x+y)^2-y^2}\ ;\ \text{at } (-1,1),\ \frac{dy}{dx}=-1$$

$y-1=-1(x+1)\Rightarrow y=-x$

or $x+y=0.$

4. (a) $9x^2+4y^2=36\ ,\ (\sqrt{2},\frac{3}{2}\sqrt{2})$

$$18x+8y\frac{dy}{dx}=0,$$

$$\frac{dy}{dx}=-\frac{9x}{4y}\ ;\ \text{at } (\sqrt{2},\tfrac{3}{2}\sqrt{2}),\ \frac{dy}{dx}=-\frac{3}{2}$$

(b) $4y^2=36-9x^2,\ (\sqrt{2},\frac{3}{2}\sqrt{2})$

$y=\pm\frac{1}{2}\sqrt{36-9x^2}$

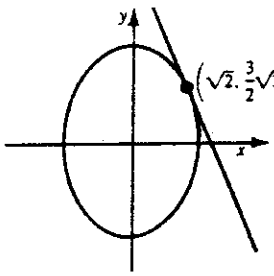
Since $y>0$, $y=\frac{1}{2}\sqrt{36-9x^2}$

$$\frac{dy}{dx}=-\frac{18x}{4\sqrt{36-9x^2}}$$

when $x=\sqrt{2}$, $\frac{dy}{dx}=-\frac{3}{2}$

(c) $y-\frac{3}{2}\sqrt{2}=-\frac{3}{2}(x-\sqrt{2}) \Rightarrow y=-\frac{3}{2}x+3\sqrt{2}$ or $3x+2y-6\sqrt{2}=0$.

(d)

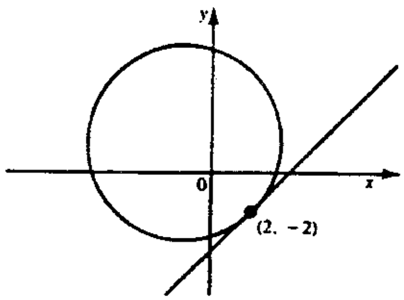


5. (a) $x^2+y^2+2x-4y-20=0$, $(2,-2)$

$2x+2y\frac{dy}{dx}+2-4\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}(2y-4)=-2-2x \Rightarrow \frac{dy}{dx}=-\frac{1+x}{y-2}$

At $(2,-2)$, $\frac{dy}{dx}=\frac{3}{4}$; $y+2=\frac{3}{4}(x-2) \Rightarrow y=\frac{3}{4}x-\frac{14}{4}$ or $3x-4y-14=0$.

(b)



6. $2(x^2+y^2)^2=25(x^2-y^2)$

(a) $4(x^2+y^2)(2x+2y\frac{dy}{dx})=25(2x-2y\frac{dy}{dx}) \Rightarrow 8x(x^2+y^2)+\frac{dy}{dx}8y(x^2+y^2)=50x-50y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(8y(x^2+y^2)+50y)=50x-8x(x^2+y^2) \Rightarrow \frac{dy}{dx}=\frac{x[25-4(x^2+y^2)]}{y[25+4(x^2+y^2)]}$

(b) At $(-3,1)$, $\frac{dy}{dx}=\frac{-3[25-4[(-3)^2+1]]}{25+4[(-3)^2+1]}=\frac{9}{13}$; $y-1=\frac{9}{13}(x+3) \Rightarrow y=\frac{9}{13}x+\frac{40}{13}$

or $9x-13y+14=0$.

(c) Horizontal tangents $\Rightarrow \frac{dy}{dx}=0$

$x[25-4(x^2+y^2)]=0 \Rightarrow x=0$ or $25-4x^2-4y^2=0$. Since $y=0$ when $x=0$, there is no horizontal tangent when $x=0$ (division by 0 in y').

Use $4x^2+4y^2=25$ (1) and $2(x^2+y^2)^2=25(x^2-y^2)$ (2).

From (1): $x^2+y^2=\frac{25}{4}$ (3) and $x^2=\frac{25-4y^2}{4}$ (4)

Substitute (3) and (4) in (2) : $2(\frac{25}{4})^2=25(\frac{25-4y^4}{4}-y^2) \Rightarrow \frac{25}{32}=(\frac{25-4y^2-4y^2}{4})$

$\Rightarrow 25=8(25-8y^2) \Rightarrow y^2=\frac{100}{64} \Rightarrow y=\pm\frac{5}{4}$

Using (4) : when $y=\pm\frac{5}{4}$, $x^2=\frac{25-4(\frac{5}{4})^2}{4}=\frac{75}{16}$

So the horizontal tangents occur at $(\pm\frac{5\sqrt{3}}{4}, \pm\frac{5}{4})$.

7. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$

(a) $\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}} = -\sqrt[3]{\frac{y}{x}}$

(b) $(\frac{1}{8}, \frac{3\sqrt{3}}{8})$; $\frac{dy}{dx} = -\sqrt[3]{\frac{\frac{3\sqrt{3}}{8}}{\frac{1}{8}}} = -\sqrt{3}$; $y - \frac{3\sqrt{3}}{8} = -\sqrt{3}(x - \frac{1}{8}) \Rightarrow y = -\sqrt{3}x + \frac{\sqrt{3}}{2}$ or

$2\sqrt{3}x + 2y - \sqrt{3} = 0.$

(c) Tangent slope of 1 $\Rightarrow \frac{dy}{dx} = 1$: $-\sqrt[3]{\frac{x}{y}} = 1 \Rightarrow \frac{y}{x} = -1 \Rightarrow y = -x$

Must find intersection point between $y = -x$ and $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$. Let $y = -x$

$x^{\frac{2}{3}} + (-x)^{\frac{2}{3}} = 1 \Rightarrow x^{\frac{2}{3}} + x^{\frac{2}{3}} = 1 \Rightarrow x^{\frac{2}{3}} = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{8}} = \pm\frac{\sqrt{2}}{4}$. So $y = \mp\frac{\sqrt{2}}{4}$.

Therefore the curve has tangents with slope 1 at $(\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4})$ and $(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})$.

8. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Show equation of tangent line at (x_0, y_0) is $\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$.

$b^2x^2 + a^2y^2 = a^2b^2 \Rightarrow 2b^2x + 2a^2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{b^2x}{a^2y}$; at (x_0, y_0), $\frac{dy}{dx} = -\frac{x_0b^2}{y_0a^2}$

Equation of tangent : $y - y_0 = -\frac{x_0b^2}{y_0a^2}(x - x_0) \Rightarrow yy_0a^2 - y_0^2a^2 = -xx_0b^2 + x_0^2b^2$

$\Rightarrow yy_0a^2 + xx_0b^2 = y_0^2a^2 + x_0^2b^2 \Rightarrow \frac{x_0x}{a^2} + \frac{y_0y}{b^2} = \frac{y_0^2}{b^2} + \frac{x_0^2}{a^2} \Rightarrow \frac{x_0x}{a^2} + \frac{y_0y}{b^2} = 1$

since (x_0, y_0) lies on the ellipse.

9. $x[f(x)]^3 + x^2f(x) = 3$, $f(2) = 1$, find $f'(2)$

$3x[f(x)]^2f'(x) + [f(x)]^3 + x^2f'(x) + 2xf(x) = 0 \Rightarrow f'(x)[3x[f(x)]^2 + x^2] = -2xf(x) - [f(x)]^3$

$\Rightarrow f'(x) = -\frac{2xf(x) + [f(x)]^3}{3x[f(x)]^2 + x^2}$: $f'(2) = -\frac{2(2)(1) + 1^3}{3(2)(1) + 2^2} = -\frac{1}{2}$

10. Let the equation of the circle be $(x-x_0)^2+(y-y_0)^2=r^2$. So the centre of the circle is (x_0, y_0). The slope of the line from the centre (x_0, y_0) to a point (x, y) on the circle is $m=\frac{y-y_0}{x-x_0}$. The slopes of the tangents to the circle are given by : $2(x-x_0)+2(y-y_0)\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x-x_0}{y-y_0}$. Since $\frac{dy}{dx}$ is the negative reciprocal of the slope of the line, the tangent to any point will be perpendicular to the radius.

11. $x^2-y^2=k$, $xy=c$

$2x-2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{x}{y}$ $\qquad$ $x\frac{dy}{dx}+y=0 \Rightarrow \frac{dy}{dx}=-\frac{y}{x}$

Since the derivatives are the negative reciprocals of each other, at any intersection point the tangents will be perpendicular.

Exercise 2.8

1. (a) $f(x) = x^5 - 4x^2 + 1$; $f'(x) = 5x^4 - 8x$; $f''(x) = 20x^3 - 8$

(b) $g(x) = 7x^4 + 12x^3 - 4x + 8$; $f'(x) = 28x^3 + 36x^2 - 4$; $f''(x) = 84x^2 + 72x$

(c) $f(t) = 2t - \frac{1}{t+1}$; $f'(t) = 2 + \frac{1}{(t+1)^2}$; $f''(t) = -\frac{2}{(1+t)^3}$

(d) $g(t) = 4t^{-\frac{1}{2}}$; $g'(t) = -2t^{-\frac{3}{2}}$; $g''(t) = 3t^{-\frac{5}{2}}$

(e) $y = (2x+1)^8$; $y' = 16(2x+1)^7$; $y'' = 224(2x+1)^6$

(f) $y = t^3 + t^{-3}$; $y' = 3t^2 - 3t^{-4}$; $y'' = 6t + 12t^{-5}$

(g) $y = (x^2+1)^{\frac{1}{2}}$; $y' = x(x^2+1)^{-\frac{1}{2}}$; $y'' = (x^2+1)^{-\frac{1}{2}} - x^2(x^2+1)^{-\frac{3}{2}}$

$= (x^2+1)^{-\frac{3}{2}}(x^2+1-x^2) \quad = (x^2+1)^{-\frac{3}{2}}$

(h) $y = \frac{t}{t-1}$; $y' = \frac{t-1-t}{(t-1)^2} = -\frac{1}{(t-1)^2}$; $y'' = \frac{2}{(t-1)^3}$

2. (a) $f(x) = 1 - 12x + 4x^2 - x^3$; $f'(x) = -12 + 8x - 3x^2$; $f''(x) = 8 - 6x$; $f'''(x) = -6$

(b) $f(x) = \frac{1}{x^5}$; $f'(x) = -\frac{5}{x^6}$; $f''(x) = \frac{30}{x^7}$; $f'''(x) = -\frac{210}{x^8}$

(c) $y = \frac{3}{(4-x)^2}$; $y' = \frac{6}{(4-x)^3}$; $y'' = \frac{18}{(4-x)^4}$; $y''' = \frac{72}{(4-x)^5}$

(d) $y = (1+2x)^{\frac{1}{2}}$; $y' = (1+2x)^{-\frac{1}{2}}$; $y'' = -(1+2x)^{-\frac{3}{2}}$; $y''' = 3(1+2x)^{-\frac{5}{2}}$

3. $y = x^5 + x^4 + x^3 + x^2 + x + 1$; $y' = 5x^4 + 4x^3 + 3x^2 + 2x + 1$;
$y'' = 20x^3 + 12x^2 + 6x + 2$; $y''' = 60x^2 + 24x + 6$; $y^{(4)} = 120x + 24$; $y^{(5)} = 120$; $y^{(6)} = 0$

4. $f(x) = (1+x^3)^{\frac{1}{2}}$; $f'(x) = \frac{3}{2}x^2(1+x^3)^{-\frac{1}{2}}$; $f''(x) = 3x(1+x^3)^{-\frac{1}{2}} - \frac{9}{4}x^4(1+x^3)^{-\frac{3}{2}}$
$f''(2) = 3(2)(1+2^3)^{-\frac{1}{2}} - \frac{9}{4}(2)^4(1+2^3)^{-\frac{3}{2}} = \frac{2}{3}$

5. $g(x) = (3x+4)^{-\frac{1}{2}}$; $g'(x) = -\frac{3}{2}(3x+4)^{-\frac{3}{2}}$; $g''(x) = \frac{27}{4}(3x+4)^{-\frac{5}{2}}$; $g'''(x) = -\frac{405}{8}(3x+4)^{-\frac{7}{2}}$
$g'''(4) = -\frac{405}{8}[3(4)+4]^{-\frac{7}{2}} = -\frac{405}{131\,072}$

6. $f(x) = x^n$: $f'(x) = nx^{n-1}$, $f''(x) = n(n-1)x^{n-2}$, $f'''(x) = n(n-1)(n-2)x^{n-3}, \ldots$
So $f^{(n)}(x) = n(n-1)(n-2) \cdot \cdots \cdot 2 \cdot 1\,x^0 = n!$

7. (a) $x^4+y^4=1$; $4x^3+4y^3y'=0 \Rightarrow y'=-\dfrac{x^3}{y^3}$, $y''=-\dfrac{3x^2y^3-3x^3y^2y'}{y^6}$

$$=-\frac{3x^3y^2-3x^2y^2(-\frac{x^3}{y^3})}{y^6}=-\frac{3x^2y^4-3x^6}{y^7}=-\frac{3x^2(y^4+x^4)}{y^7}=-\frac{3x^2}{y^7}$$

(b) $x^2-y^2=1$; $2x-2yy'=0 \Rightarrow y'=\frac{x}{y}$; $y''=\dfrac{y-xy'}{y^2}=\dfrac{y-x(\frac{x}{y})}{y^2}=\dfrac{y^2-x^2}{y^3}=-\dfrac{1}{y^3}$

(c) $x^3+y^3=6xy$; $3x^2+3y^2y'=6xy'+6y \Rightarrow y'(6x-3y^2)=3x^2-6y \Rightarrow y'=\dfrac{x^2-2y}{2x-y^2}$

$$y''=\frac{(2x-y^2)(2x-2y')-(x^2-2y)(2-2yy')}{(2x-y^2)^2}$$

$$=\frac{4x^2-4xy'-2xy^2+2y^2y'-2x^2+2x^2yy'+4y-4y^2y'}{(2x-y^2)^2}$$

$$=\frac{2x^2+4y-2xy^2+y'(2x^2y-2y^2-4x)}{(2x-y^2)^2}=\frac{2x^2+4y-2xy^2+\frac{(x^2-2y)(2x^2y-2y^2-4x)}{(2x-y^2)}}{(2x-y^2)^2}$$

$$=\frac{(2x^2+4y-2xy^2)(2x-y^2)+(x^2-2y)(2x^2y-2y^2-4x)}{(2x-y^2)^3}$$

$$=\frac{4x^3-2x^2y^2+8xy-4y^3-4x^2y^2+2xy^4+2x^4y-2x^2y^2-4x^3-4x^2y^2+4y^3+8xy}{(2x-y^2)^3}$$

$$=\frac{-12x^2y^2+16xy+2xy^4+2x^4y}{(2x-y^2)^3}=\frac{-12x^2y^2+16xy+2xy(x^3+y^3)}{(2x-y^2)^3}$$

$$=\frac{-12x^2y^2+16xy+2xy(6xy)}{(2x-y^2)^3}=\frac{16xy}{(2x-y^2)^3}$$

8. $f(3)=33$, $f'(3)=22$, $f''(3)=8$: Let the function be ax^2+bx+c
So $ax^2+bx+c=33$, $f'(x)=2ax+b=22$, $f''(x)=2a=8 \Rightarrow a=4$.
Substituting $a=4$ and $x=3$ in the others gives, $2(4)(3)+b=22 \Rightarrow b=-2$ and $4(3)^2-2(3)+c=33 \Rightarrow c=3$. So a function that meets the given requirements is $f(x)=4x^2-2x+3$.

9. $f(x)=g(x)h(x)$
(a) $f'=hg'+gh'$; $f''=hg''+g'h'+gh''+h'g'=hg''+2h'g'+gh''$
(b) $f'''=hg'''+g''h'+2h'g''+2g'h''+gh'''+h''g'=hg'''+3h'g''+3g'h''+gh'''$

10. (a) $f(x) = |x^2 - 1|$, which can be written as $f(x) = \begin{cases} x^2 - 1 & \text{if } |x| \geq 1 \\ -x^2 + 1 & \text{if } |x| < 1 \end{cases}$

Since f(x) is a polynomial when $x > 1$, $x < -1$, $f'(x)$ exists here.

Similarly for $-1 < x < 1$, f(x) is a polynomial so $f'(x)$ exists here. This gives :

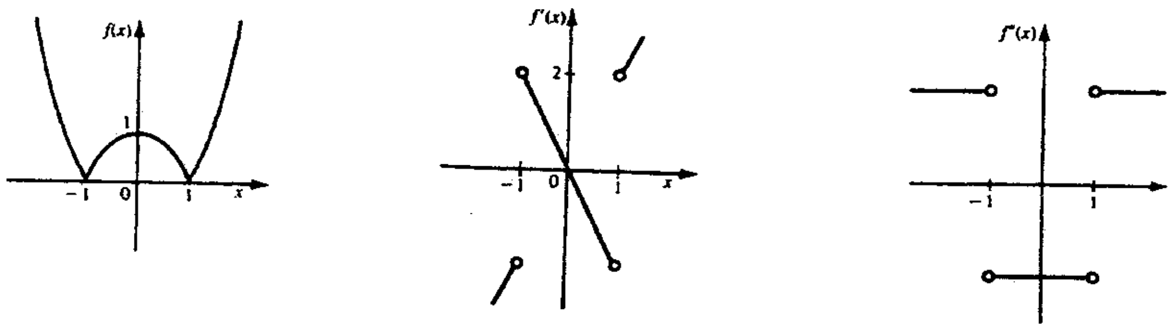
$$f'(x) = \begin{cases} 2x & \text{if } |x| > 1 \\ -2x & \text{if } |x| < 1 \end{cases}$$

We see from the graph of f that $f'(1)$ and $f'(-1)$ do not exist, so the domain of f' is $\{x | x \neq \pm 1\}$.

Since $f'(x)$ is a polynomial on $x > 1, -1 < x < 1$, and $x < -1$, $f''(x)$ exists here , so $\text{dom}(f'') = \{x | x \neq \pm 1\}$ and

$$f''(x) = \begin{cases} 2 & \text{if } |x| > 1 \\ -2 & \text{if } |x| < 1 \end{cases}$$

(b)



Review Exercise 2.9

1. (a) $f(x) = 1 - 2x + 3x^2$; $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{1 - 2(x+h) + 3(x+h)^2 - [1 - 2x + 3x^2]}{h}$$

$$= \lim_{h \to 0} \frac{1 - 2x - 2h + 3x^2 + 6xh + 3h^2 - 1 + 2x - 3x^2}{h} = \lim_{h \to 0} \frac{6xh - 2h + 3h^2}{h}$$

$$= \lim_{h \to 0} (6x - 2 + 3h) = 6x - 2$$

(b) $f(x) = x^3 + 4x$; $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{(x+h)^3 + 4(x+h) - [x^3 + 4x]}{h}$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 4x + 4h - x^3 - 4x}{h} = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 + 4h}{h}$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2 + 4) = 3x^2 + 4$$

(c) $f(x) = \dfrac{x}{1-x}$; $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{\frac{x+h}{1-(x+h)} - \frac{x}{1-x}}{h}$

$$= \lim_{h \to 0} \frac{x - x^2 - xh + h - x + x^2 + xh}{h(1-x-h)(1-x)} = \lim_{h \to 0} \frac{1}{(1-x-h)(1-x)} = \frac{1}{(1-x)^2}$$

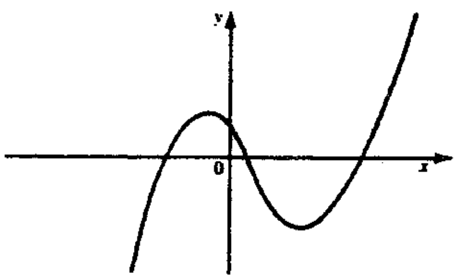
(d) $f(x) = \sqrt{2x+1}$; $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \times \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

$$= \lim_{h \to 0} \frac{2x + 2h + 1 - 2x - 1)}{h\left[\sqrt{2(x+h)+1} + \sqrt{2x+1}\right]} = \lim_{h \to 0} \frac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

2. $\lim_{h \to 0} \dfrac{(1+h)^4 - 1}{h}$; $a = 1$, $f(x) = x^4$

3. By drawing tangents to the graph in the text, we get the following:



4. (a) $y = 12x^3+8x-1$; $y' = 36x^2+8$ (b) $y = 2x^{\pi+1}$; $y' = 2(\pi+1)x^{\pi}$

(c) $y = 2x-\frac{3}{x}$; $y' = 2+\frac{3}{x^2}$ (d) $y = \sqrt[5]{x^6}$; $y' = \frac{6}{5}\sqrt[5]{x}$

(e) $y = \sqrt{x}(5-\sqrt{x}) = 5\sqrt{x}-x$; $y' = \frac{5}{2\sqrt{x}}-1$

(f) $y = \frac{x^2-2x}{\sqrt{x}} = \sqrt{x^3}-2\sqrt{x}$; $y' = \frac{3}{2}\sqrt{x}-\frac{1}{\sqrt{x}} = \frac{3x-2}{2\sqrt{x}}$

(g) $y = \frac{2x-1}{1+3x}$; $y' = \frac{2(1+3x)-3(2x-1)}{(1+3x)^2} = \frac{2+6x-6x+3}{(1+3x)^2} = \frac{5}{(1+3x)^2}$

(h) $y = (2x^3-1)^7$; $y' = 7(2x^3-1)^6(6x^2) = 42x^2(2x^3-1)^6$

(i) $f(x) = (x^2+x)\sqrt{1-x^2}$; $f'(x) = \frac{x^2+x}{2\sqrt{1-x^2}}(-2x) + (2x+1)\sqrt{1-x^2}$

$= \frac{1}{\sqrt{1-x^2}}[(-x)(x^2+x)+(2x+1)(1-x^2)] = \frac{1}{\sqrt{1-x^2}}[-x^3-x^2+2x-2x^3+1-x^2]$

$= \frac{-3x^3-2x^2+2x+1}{\sqrt{1-x^2}}$

(j) $g(x) = \frac{3x^2+1}{2-x}$; $g'(x) = \frac{6x(2-x)+(3x^2+1)}{(2-x)^2} = \frac{12x-6x^2+3x^2+1}{(2-x)^2}$

$= \frac{-3x^2+12x+1}{(2-x)^2}$

(k) $h(x) = \frac{1}{\sqrt[3]{2x^4-1}} = (2x^4-1)^{-\frac{1}{3}}$; $h'(x) = -\frac{1}{3}(2x^4-1)^{-\frac{4}{3}}(8x^3) = \frac{-8x^3}{3\sqrt[3]{(2x^4-1)^4}}$

(l) $F(x) = (x^4+1)^3(1-2x)$; $F'(x) = -2(x^4+1)^3+12x^3(x^4+1)^2(1-2x)$

$= 2(x^4+1)^2[-x^4-1+6x^3-12x^4] = 2(x^4+1)^2(-13x^4+6x^3-1)$

(m) $f(t) = \frac{t}{\sqrt{1+2t}} = t(1+2t)^{-\frac{1}{2}}$; $f'(t) = -t(1+2t)^{-\frac{3}{2}}+(1+2t)^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{(1+2t)^3}}(-t+1+2t) = \frac{1+t}{\sqrt{(1+2t)^3}}$

(n) $g(t) = \left(\frac{t+1}{t+2}\right)^4$; $g'(t) = 4\left(\frac{t+1}{t+2}\right)^3\left(\frac{t+2-t-1}{(t+2)^2}\right) = \frac{4(t+1)^3}{(t+2)^5}$

(o) $R(u) = \sqrt[4]{u+1}-\frac{2}{u^2} = (u+1)^{\frac{1}{4}}-2u^{-2}$;

$R'(u) = \frac{1}{4}(u+1)^{-\frac{3}{4}}+4u^{-3} = \frac{1}{4\sqrt[4]{(u+1)^3}} + \frac{4}{u^3}$

(p) $S(v) = \sqrt{v-(v^2-8)^5} = [v-(v^2-8)^5]^{\frac{1}{2}}$; $S'(v) = \frac{1}{2}[v-(v^2-8)^5]^{-\frac{1}{2}}[1-10v(v^2-8)^4]$

$= \dfrac{1-10v(v^2-8)^4}{2\sqrt{v-(v^2-8)^5}}$

(q) $M(z) = \sqrt{\dfrac{1+z}{1+z^2}} = \left(\dfrac{1+z}{1+z^2}\right)^{\frac{1}{2}}$; $M'(z) = \frac{1}{2}\left(\dfrac{1+z}{1+z^2}\right)^{-\frac{1}{2}}\left(\dfrac{1+z^2-2z(1+z)}{(1+z^2)^2}\right)$

$= \frac{1}{2}\left(\dfrac{1+z^2}{1+z}\right)^{\frac{1}{2}}\left(\dfrac{1-2z-z^2}{(1+z^2)^2}\right) = \dfrac{1-2z-z^2}{2\sqrt{1+z}\sqrt{(1+z^2)^3}}$

(r) $F(y) = \dfrac{1}{2+\frac{3}{y}} = (2+\frac{3}{y})^{-1}$; $F'(y) = -(2+\frac{3}{y})^{-2}(-\frac{3}{y^2}) = \dfrac{3}{y^2(2+\frac{3}{y})^2} = \dfrac{3}{(2y+3)^2}$

5. (a) $f(x) = \dfrac{2x-1}{x^2-5}$; $f'(x) = \dfrac{2(x^2-5)-2x(2x-1)}{(x^2-5)^2}$

$= \dfrac{2x^2-10-4x^2+2x}{(x^2-5)^2} = \dfrac{-2x^2+2x-10}{(x^2-5)^2}$

Domain of f, $D = \{x \mid x^2-5 \neq 0\} = \{x \mid x \neq \pm\sqrt{5}\}$. Domain of f′, $D' = \{x \mid x \neq \pm\sqrt{5}\}$

(b) $f(x) = \sqrt{x^2-x-6}$; $f'(x) = \dfrac{2x-1}{2\sqrt{x^2-x-6}}$; Domain of f, $D = \{x \mid x^2-x-6 \geq 0\}$

$= \{x \mid (x-3)(x+2) \geq 0\} = \{x \mid x-3 \geq 0 \text{ and } x+2 \geq 0 \text{ or } x-3 \leq 0 \text{ and } x+2 \leq 0\}$

$= \{x \mid x \geq 3 \text{ and } x \geq -2 \text{ or } x \leq 3 \text{ and } x \leq -2\} = \{x \mid x \geq 3 \text{ or } x \leq -2\}$.

Domain of f′, $D' = \{x \mid x^2-x-6 > 0\} = \{x \mid x > 3 \text{ or } x < -2\}$

6. $y = u^2-u^3+2u^4$, $u = \dfrac{x}{2x-1}$; $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (2u-3u^2+8u^3)\left(\dfrac{2x-1-2x}{(2x-1)^2}\right)$

$= \dfrac{-8u^3+3u^2-2u}{(2x-1)^2}$: When $x=1$, $u = \dfrac{1}{2-1} = 1$. $\left[\dfrac{dy}{dx}\right]_{x=1} = \dfrac{-8+3-2}{(2-1)^2} = -7$

7. (a) $x^4+y^4 = 1$; $4x^3+4y^3\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x^3}{y^3}$

(b) $x^2-x^2y+y^2 = 1$; $2x-x^2\dfrac{dy}{dx}-2xy+2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx}(2y-x^2) = 2xy-2x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2xy-2x}{2y-x^2}$

(c) $2x^2y^2 = x^3+y^3$; $2x^2(2y)\dfrac{dy}{dx}+4xy^2 = 3x^2+3y^2\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx}(4x^2y-3y^2) = 3x^2-4xy^2$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{3x^2-4xy^2}{4x^2y-3y^2}$

(d) $y\sqrt{x-1}+x\sqrt{y-1} = xy$; $\frac{y}{2\sqrt{x-1}}+\frac{dy}{dx}\sqrt{x-1} + \frac{x}{2\sqrt{y-1}}\frac{dy}{dx}+\sqrt{y-1} = x\frac{dy}{dx}+y$

$\Rightarrow \frac{dy}{dx}\left(\sqrt{x-1} + \frac{x}{2\sqrt{y-1}} - x\right) = y-\frac{y}{2\sqrt{x-1}} - \sqrt{y-1} \Rightarrow$

$$\frac{dy}{dx} = \frac{y-\frac{y}{2\sqrt{x-1}}-\sqrt{y-1}}{\sqrt{x-1}+\frac{x}{2\sqrt{y-1}}-x}$$

8. (a) $y = 4x^5-\frac{1}{2}x^4+3x^2$; $y' = 20x^4-2x^3+6x$; $y'' = 80x^3-6x^2+6$

(b) $y = \sqrt{3x+1}$; $y' = \frac{3}{2}(3x+1)^{-\frac{1}{2}} = \frac{3}{2\sqrt{3x+1}}$; $y'' = -\frac{9}{4}(3x+1)^{-\frac{3}{2}} = \frac{-9}{4\sqrt{(3x+1)^3}}$

(c) $y = \frac{t-1}{t+1}$; $y' = \frac{t+1-t+1}{(t+1)^2} = \frac{2}{(t+1)^2}$; $y'' = \frac{-4}{(t+1)^3}$

(d) $x^2+y^2 = 16$; $2x+2yy' = 0 \Rightarrow y' = -\frac{x}{y}$; $y'' = -\frac{y-x\frac{dy}{dx}}{y^2} \Rightarrow y'' = -\frac{y-x(-\frac{x}{y})}{y^2}$

$\Rightarrow y'' = -\frac{y^2+x^2}{y^3} \Rightarrow y'' = -\frac{16}{y^3}$

9. (a) $y = x^2-2x+5$, $(-1,8)$

$y' = 2x-2$; when $x=-1$, $y' = 2(-2)-2 = -4$; $y-8 = -4(x+1) \Rightarrow y = -4x+4$ or $4x+y-4 = 0$.

(b) $y = \frac{2}{1-x}$, $(2,-2)$;

$y' = \frac{2}{(1-x)^2}$; when $x=2$, $y' = \frac{2}{(1-2)^2} = 2$; $y+2 = 2(x-2) \Rightarrow y = 2x-6$ or

$2x-y-6 = 0$.

(c) $y = \frac{1}{\sqrt{x^5}}$, $(2,\frac{1}{4\sqrt{2}})$

$y' = \frac{-5}{2\sqrt{x^7}}$; when $x=2$, $y' = \frac{-5}{2\sqrt{2^7}} = \frac{-5\sqrt{2}}{32}$; $y-\frac{1}{4\sqrt{2}} = -\frac{5\sqrt{2}}{32}(x-2)$

$\Rightarrow y = -\frac{5\sqrt{2}}{32}x+\frac{5\sqrt{2}}{16}+\frac{1}{4\sqrt{2}} \Rightarrow y = -\frac{5\sqrt{2}}{32}x+\frac{7\sqrt{2}}{16}$ or $5\sqrt{2}x+32y-14\sqrt{2} = 0$.

(d) $y = x\sqrt{x^2+5}$, $(-2,-6)$

$y' = \frac{1}{2}x(x^2+5)^{-\frac{1}{2}}(2x)+\sqrt{x^2+5} \Rightarrow y' = \frac{x^2}{\sqrt{x^2+5}} + \sqrt{x^2+5}$;

when $x=-2$, $y' = \dfrac{(-2)^2}{\sqrt{(-2)^2+5}} + \sqrt{(-2)^2+5} = \frac{4}{3} + 3 = \frac{13}{3}$; $y+6 = \frac{13}{3}(x+2)$

$\Rightarrow y = \frac{13}{3}x + \frac{8}{3}$ or $13x - 3y + 8 = 0$.

(e) $(x-1)^2 + (y+2)^2 = 25$, $(-2,2)$

$2(x-1) + 2(y+2)y' = 0 \Rightarrow y' = -\dfrac{x-1}{y+2}$; when $x=-2$, $y' = -\dfrac{-2-1}{2+2} = \frac{3}{4}$;

$y-2 = \frac{3}{4}(x+2) \Rightarrow y = \frac{3}{4}x + \frac{7}{2}$ or $3x - 4y + 14 = 0$.

(f) $x^3 + y^3 = 9xy$, $(2,4)$

$3x^2 + 3y^2y' = 9xy' + 9y \Rightarrow y'(3y^2 - 9x) = 9y - 3x^2 \Rightarrow y' = \dfrac{3y - x^2}{y^2 - 3x}$;

when $x=2$, $y=4$, $y' = \dfrac{3(4) - 2^2}{4^2 - 3(2)} = \frac{8}{10} = \frac{4}{5}$; $y - 4 = \frac{4}{5}(x-2) \Rightarrow y = \frac{4}{5}x + \frac{12}{5}$ or

$4x - 5y + 12 = 0$.

10. $h = 550 - 5t^2$; velocity $v(t) = h'(t) = -10t$

$v(1) = -10$ m/s $\quad$ $v(2) = -20$ m/s $\quad$ $v(5) = -50$ m/s

11. $y = 2x^2 - 3x + 6$, tangent $\parallel$ to $7x + y = 1 \Rightarrow y = -7x + 1$; So tangent slope is -7. $y' = 4x - 3 = -7$ when $x = -1$. At $x = -1$, $y = 2(-1)^2 - 3(-1) + 6 = 11$. So the point on the curve where the tangent has slope -7 is $(-1, 11)$.

12. $y = \dfrac{1}{2x-1}$, tangent $\perp$ to $x - 2y = 1 \Rightarrow y = \frac{1}{2}x - \frac{1}{2}$; Slope of line is $\frac{1}{2}$, so slope of tangent is -2. $y' = \dfrac{-2}{(2x-1)^2} = -2$ when $x=0$ or $x=1$. When $x=0$, $y = \dfrac{1}{0-1}$ $= -1$; $x=1$, $y = \dfrac{1}{2-1} = 1$. So the points are $(0,-1)$ and $(1,1)$.

13. $(2,-3)$, tangent to $y = x^2 + x$; $y' = 2x + 1$. Let the x-coordinate of the tangent point be a, so the y value is $a^2 + a$. So the slope of the tangent is given by $m = \dfrac{a^2 + a + 3}{a - 2}$ [slope of line through $(a, a^2 + a)$ and $(2, -3)$] and by $y' = 2a + 1$.

So $2a + 1 = \dfrac{a^2 + a + 3}{a - 2} \Rightarrow 2a^2 - 4a + a - 2 = a^2 + a + 3 \Rightarrow a^2 - 4a - 5 = (a-5)(a+1)$ $= 0$, so $a = 5$, $a = -1$. When $x = 5$, $y = 5^2 + 5 = 30$ and $y' = 2(5) + 1 = 11$, so $y - 30 = 11(x - 5) \Rightarrow y = 11x - 25$ or $11x - y - 25 = 0$.

When $x = -1$, $y = (-1)^2 - 1 = 0$ and $y' = 2(-1) + 1 = -1$, so $y = -(x+1) \Rightarrow$ $y = -x - 1$ or $x + y + 1 = 0$.

14. $f(3) = 4$, $f'(3) = -1$, $f'(6) = 5$, $g(3) = 6$, and $g'(3) = 2$

(a) $(fg)'(3) = f(3)g'(3) + f'(3)g(3) = (4)(2) + (-1)(6) = 2$

(b) $(\frac{f}{g})'(3) = \dfrac{f'(3)g(3) - f(3)g'(3)}{[g(3)]^2} = \dfrac{(-1)(6) - (4)(2)}{(6)^2} = -\dfrac{7}{18}$

(c) $(f \circ g)'(3) = f'(g(3))g'(3) = f'(6)(2) = (5)(2) = 10$

15. (a) $f(x) = x^2 g(x)$; $f'(x) = x^2 g'(x) + 2xg(x)$

(b) $f(x) = \dfrac{g(x)}{\sqrt{x}}$; $f'(x) = \dfrac{\sqrt{x}\, g'(x) - \frac{1}{2\sqrt{x}} g(x)}{x} = \dfrac{2xg'(x) - g(x)}{2\sqrt{x^3}}$

(c) $f(x) = g(\frac{1}{x})$; $f'(x) = -\frac{1}{x^2} g'(\frac{1}{x})$

(d) $f(x) = \sqrt{g(\sqrt{x})}$; $f'(x) = \dfrac{1}{2\sqrt{g(\sqrt{x})}} g'(\sqrt{x}) \dfrac{1}{2\sqrt{x}} = \dfrac{g'(\sqrt{x})}{4\sqrt{xg(\sqrt{x})}}$

16. $f(x) = g(g(x))$; $f'(x) = g'(g(x))g'(x)$; $f''(x) = g'(g(x))g''(x) + g''(g(x))[g'(x)]^2$

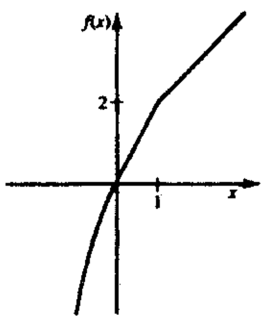
17. (a) The only two places f might not be differentiable are $x=0$, $x=1$ since f is a polynomial everywhere else. f(x) will be differentiable at 0 if

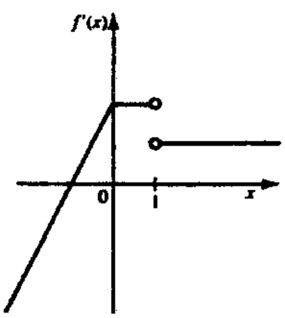
$\left[\frac{d}{dx}(2x - x^2)\right]_{x=0} = \left[\frac{d}{dx}(2x)\right]_{x=0}$; $\left[\frac{d}{dx}(2x - x^2)\right]_{x=0} = [(2-2x)]_{x=0} = 2$,

$\left[\frac{d}{dx}(2x)\right]_{x=0} = 2$; so f is differentiable at $x=0$.

From the graph of f we see that f is not differentiable at 1.

(b) $f(x) = \begin{cases} 2 - 2x & \text{if } x \le 0 \\ 2 & \text{if } 0 < x < 1 \\ 1 & \text{if } x > 1 \end{cases}$





Chapter 2 Test

1. (a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

(b) (i) $f(x) = x^2 - 7x + 4$; $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 7(x+h) + 4 - [x^2 - 7x + 4]}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 7x - 7h + 4 - x^2 + 7x - 4}{h} = \lim_{h \to 0} \frac{2xh + h^2 - 7h}{h}$

$= \lim_{h \to 0} (2x + h - 7) = 2x - 7$

(ii) $f(x) = \frac{1}{2x+1}$; $f'(x) = \lim_{h \to 0} \frac{\frac{1}{2(x+h)+1} - \frac{1}{2x+1}}{h} = \lim_{h \to 0} \frac{2x + 1 - 2x - 1 - 2h}{h(2x+1)(2x+2h+1)}$

$= \lim_{h \to 0} \frac{-2}{(2x+1)(2x+2h+1)} = \frac{-2}{(2x+1)^2}$

2. (a) $f(x) = \sqrt[3]{x^2}$; $f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

(b) $f(x) = \frac{x^2+3}{2x-1}$; $f'(x) = \frac{2x(2x-1) - 2(x^2+3)}{(2x-1)^2} = \frac{2x^2 - 2x - 6}{(2x-1)^2}$

(c) $f(x) = (x^2-1)^4(2x+1)^3$; $f'(x) = 6(2x+1)^2(x^2-1)^4 + 8x(x^2-1)^3(2x+1)^3$

$= 2(x^2-1)^3(2x+1)^2[3(x^2-1) + 4x(2x+1)] = 2(x^2-1)^3(2x+1)^2(11x^2 + 4x - 3)$

(d) $f(x) = (x + \sqrt{x^4 - 2x + 1})^7$; $f'(x) = 7(x + \sqrt{x^4 - 2x + 1})^6(1 + \frac{1}{2\sqrt{x^4 - 2x + 1}}(4x^3 - 2))$

$= 7(1 + \frac{2x^3 - 1}{\sqrt{x^4 - 2x + 1}})(x + \sqrt{x^4 - 2x + 1})^6$

3. $3xy = x^3 + y^3$

(a) $3x\frac{dy}{dx} + 3y = 3x^2 + 3y^2\frac{dy}{dx} \Rightarrow \frac{dy}{dx}(3x - 3y^2) = 3x^2 - 3y \Rightarrow \frac{dy}{dx} = \frac{x^2 - y}{x - y^2}$

(b) $(\frac{2}{3}, \frac{4}{3})$; At this point, $\frac{dy}{dx} = \frac{(\frac{2}{3})^2 - \frac{4}{3}}{\frac{2}{3} - (\frac{4}{3})^2} = \frac{\frac{4}{9} - \frac{12}{9}}{\frac{6}{9} - \frac{16}{9}} = \frac{4}{5}$

$y - \frac{4}{3} = \frac{4}{5}(x - \frac{2}{3}) \Rightarrow y = \frac{4}{5}x + \frac{12}{15}$ or $4x - 5y + 4 = 0.$

4. $y = \frac{1}{(3-2x)^2} = (3-2x)^{-2}$; $y' = 4(3-2x)^{-3}$; $y'' = 24(3-2x)^{-4}$; $y''' = \frac{192}{(3-2x)^5}$

5. $y = \sqrt{2x-1}$, tangent $\parallel$ to $x - 3y = 16 \Rightarrow y = \frac{1}{3}x - \frac{16}{3}$; So the slope of the tangent is $\frac{1}{3}$. $y' = \frac{1}{\sqrt{2x-1}} = \frac{1}{3}$ when $x=5 \Rightarrow y = 3$. So the point is $(5,3)$

6. (a) $g(x) = f(x^6)$; $g'(x) = 6x^5f'(x^6)$ (b) $h(x) = [f(x)]^6$; $h'(x) = 6f'(x)[f(x)]^5$

(c) $F(x) = \frac{x^2}{f(x)}$; $F'(x) = \frac{2xf(x) - x^2f'(x)}{[f(x)]^2}$