16. (a) The demand function is $p(x) = 400 - \frac{20}{100}(x - 800) = 560 - \frac{1}{5}x$.

(b) The revenue function is $R(x) = x\,p(x) = 560x - \frac{1}{5}x^2$. $R'(x) = 560 - \frac{2}{5}x = 0$ when $x = 1400$. $p(1400) = 400 - \frac{1}{5}(600) = 280$. Thus the price that will maximize revenue is \$280.

REVIEW AND PREVIEW TO CHAPTER 8

EXERCISE 1.

1. (a) $(-3)^5 = -243$
   (b) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$
   (c) $2^{-3}5^4 = \frac{625}{8}$
   (d) $3^{-2} - (1.7)^0 = \frac{1}{9} - 1 = -\frac{8}{9}$
   (e) $36^{\frac{1}{2}} = 6$
   (f) $(-64)^{\frac{1}{3}} = -4$
   (g) $125^{\frac{2}{3}} = 25$
   (h) $9^{-\frac{7}{2}} = 3^{-7} = \frac{1}{2187}$

2. (a) $128 = 2^7$
   (b) $2^6 \times 8^4 = 2^6 \times (2^3)^4 = 2^6 \times 2^{12} = 2^{18}$
   (c) $(2^9)^4 = 2^{36}$
   (d) $\frac{1}{4} = 2^{-2}$
   (e) $\frac{2^{3.1}}{2^{4.6}} = 2^{-\frac{3}{2}}$
   (f) $\sqrt{2} = 2^{\frac{1}{2}}$
   (g) $4\sqrt{2} = 2^2 \times 2^{\frac{1}{2}} = 2^{\frac{5}{2}}$
   (h) $1 = 2^0$

3. (a) $(12x^2y^4)(\frac{1}{2}x^5y) = 6x^7y^5$
   (b) $(2s^3t^{-1})(\frac{1}{4}s^6)(16t^4) = 8s^9t^3$
   (c) $\frac{x^9(2x)^4}{x^3} = 16x^{10}$
   (d) $\frac{a^{-3}b^4}{a^{-5}b^5} = \frac{a^2}{b}$
   (e) $(rs)^3(2s)^{-2}(4r)^4 = 64r^7s$
   (f) $(2u^2v^3)^3(3u^3v)^{-2} = \frac{8v^7}{9}$
   (g) $\frac{(x^2y^3)^4(xy^4)^{-3}}{x^2y} = \frac{x^3}{y}$
   (h) $\left(\frac{c^4d^3}{cd^2}\right)\left(\frac{d^2}{c^3}\right)^3 = \frac{d^7}{c^6}$
   (i) $\frac{a^{-1}+b^{-1}}{(a+b)^{-1}} = (a+b)\left(\frac{1}{a}+\frac{1}{b}\right) = (a+b)\left(\frac{a+b}{ab}\right) = \frac{(a+b)^2}{ab}$
   (j) $\frac{(y^{10}z^{-5})^{\frac{1}{5}}}{(y^{-2}z^3)^{\frac{1}{3}}} = \frac{y^2z^{-1}}{y^{-\frac{2}{3}}z} = \frac{y^{\frac{8}{3}}}{z^2}$
   (k) $\frac{(9st)^{\frac{3}{2}}}{(27s^3t^{-4})^{\frac{2}{3}}} = \frac{27s^{\frac{3}{2}}t^{\frac{3}{2}}}{9s^2t^{-\frac{8}{3}}} = \frac{3t^{\frac{25}{6}}}{s^{\frac{1}{2}}}$

(l) $\left(\frac{a^{2} b^{-3}}{x^{-1} y^{2}}\right)^{3}\left(\frac{x^{-2} b^{-1}}{a^{\frac{3}{2}} y^{\frac{1}{3}}}\right)=\frac{a^{\frac{9}{2}} x}{b^{10} y^{\frac{19}{3}}}$

EXERCISE 2.

1. (a) $\log_2 64 = 6$, so $2^6 = 64$
   (b) $\log_5 1 = 0$, so $5^0 = 1$
   (c) $\log_{10} 0.01 = -2$, so $10^{-2} = 0.01$
   (d) $\log_8 4 = \frac{2}{3}$, so $8^{\frac{2}{3}} = 4$
   (e) $\log_8 512 = 3$, so $8^3 = 512$
   (f) $\log_2\left(\frac{1}{16}\right) = -4$, so $2^{-4} = \frac{1}{16}$
   (g) $\log_a b = c$, so $a^c = b$
   (h) $\log_r v = w$, so $r^w = v$

2. (a) $2^3 = 8$, so $\log_2 8 = 3$
   (b) $10^5 = 100\ 000$, so $\log_{10} 100\ 000 = 5$
   (c) $10^{-4} = 0.0001$, so $\log_{10} 0.0001 = -4$
   (d) $81^{\frac{1}{2}} = 9$, so $\log_{81} 9 = \frac{1}{2}$
   (e) $4^{-\frac{3}{2}} = 0.125$, so $\log_4 0.125 = -\frac{3}{2}$
   (f) $6^{-1} = \frac{1}{6}$, so $\log_6 \frac{1}{6} = -1$
   (g) $r^s = t$, so $\log_r t = s$
   (h) $10^m = n$, so $\log_{10} n = m$

3. (a) $\log_6 6^4 = 4 \log_6 6 = 4$
   (b) $\log_2 32 = 5$
   (c) $\log_4 64 = 3$
   (d) $\log_8 8^{17} = 17 \log_8 8 = 17$
   (e) $\log_9 9 = 1$
   (f) $\log_6 1 = 0$
   (g) $\log_3\left(\frac{1}{27}\right) = -3$
   (h) $\log_4 8 = \frac{3}{2}$
   (i) $\log_8 0.25 = -\frac{2}{3}$
   (j) $\log_9 \sqrt{3} = \frac{1}{4}$

4. (a) $\log_2 x = 10$, so $x = 2^{10} = 1024$
   (b) $\log_5 x = 4$, so $x = 5^4 = 625$
   (c) $\log_{10}(3x+5) = 2$, so $3x + 5 = 10^2 = 100$, $3x = 95$, $x = \frac{95}{3}$
   (d) $\log_3(2-x) = 3$, so $2 - x = 27$, $x = -25$
   (e) $2^{1-x} = 3$, so $1 - x = \log_2 3$, $x = 1 - \log_2 3$
   (f) $3^{2x-1} = 5$, so $2x - 1 = \log_3 5$, $x = \frac{1 + \log_3 5}{2}$
   (g) $\log_2(\log_3 x) = 4$, so $\log_3 x = 2^4 = 16$, $x = 3^{16} = 43\ 046\ 721$
   (h) $10^{5^x} = 3$, so $x = \log_5(\log_{10} 3)$

EXERCISE 3.

1. (a) $\log_2 x(x-1) = \log_2 x + \log_2(x-1)$ (b) $\log_5\left(\frac{x}{2}\right) = \log_5 x - \log_5 2$

   (c) $\log_2(AB^2) = \log_2 A + 2\log_2 B$ (d) $\log_6 \sqrt[4]{17} = \frac{1}{4}\log_6 17$

   (e) $\log_3(x\sqrt{y}) = \log_3 x + \frac{1}{2}\log_3 y$ (f) $\log_2(xy)^{10} = 10\log_2 x + 10\log_2 y$

   (g) $\log_5 \sqrt[3]{x^2+1} = \frac{1}{3}\log_5(x^2+1)$ (h) $\log_b \frac{x^2}{yz^3} = 2\log_b x - \log_b y - 3\log_b z$

   (i) $\log_{10}\frac{x^3y^4}{z^6} = 3\log_{10} x + 4\log_{10} y - 6\log_{10} z$

   (j) $\log_{10}\frac{a^2}{b^4\sqrt{c}} = 2\log_{10} a - 4\log_{10} b - \frac{1}{2}\log_{10} c$

2. (a) $\log_5\sqrt{125} = \frac{1}{2}\log_5 125 = \frac{3}{2}$

   (b) $\log_2 112 - \log_2 7 = \log_2\left(\frac{112}{7}\right) = \log_2 16 = 4$

   (c) $\log_{10} 2 + \log_{10} 5 = \log_{10} 10 = 1$

   (d) $\log_{10}\sqrt{0.1} = \frac{1}{2}\log_{10} 0.1 = -\frac{1}{2}$

   (e) $\log_4 192 - \log_4 3 = \log_4\left(\frac{192}{3}\right) = \log_4 64 = 3$

   (f) $\log_{12} 9 + \log_{12} 16 = \log_{12} 144 = 2$

3. (a) $\log_{10} 12 + \frac{1}{2}\log_{10} 7 - \log_{10} 2 = \log_{10}\left(\frac{12\sqrt{7}}{2}\right) = \log_{10}(6\sqrt{7})$

   (b) $\log_2 A + \log_2 B - 2\log_2 C = \log_2\left(\frac{AB}{C^2}\right)$

   (c) $\log_5(x^2-1) - \log_5(x-1) = \log_5\left[\frac{(x+1)(x-1)}{(x-1)}\right] = \log_5(x+1)$,

   (d) $4\log_2 x - \frac{1}{3}\log_2(x^2+1) + \log_2(x-1) = \log_2\left[\frac{x^4(x-1)}{\sqrt[3]{x^2+1}}\right]$

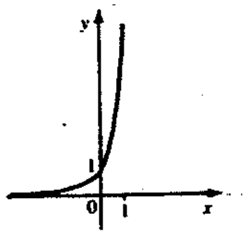
   (e) $\frac{1}{2}\left[\log_5 x + 2\log_5 y - 3\log_5 z\right] = \frac{1}{2}\log_5\left(\frac{xy^2}{z^3}\right) = \log_5\left(y\sqrt{\frac{x}{z^3}}\right)$

   (f) $\log_a b + c\log_a d - r\log_a s = \log_a\left(\frac{bd^c}{s^r}\right)$

EXERCISE 8.1

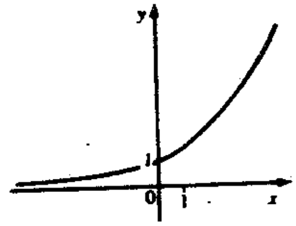
1. (a) $f(x) = 6^x$

x	f(x)
-2	$\frac{1}{36}$
-1	$\frac{1}{6}$
0	1
1	6
2	36



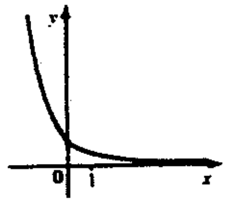
(b) $f(x) = \left(\frac{3}{2}\right)^x$

x	f(x)
-2	$\frac{4}{9}$
-1	$\frac{2}{3}$
0	1
1	$\frac{3}{2}$
2	$\frac{9}{4}$



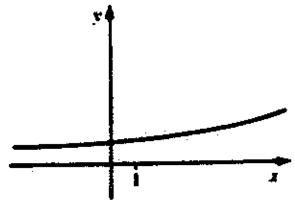
(c) $g(x) = \left(\frac{1}{4}\right)^x$

x	g(x)
-2	16
-1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$



(d) $h(x) = (1.1)^x$

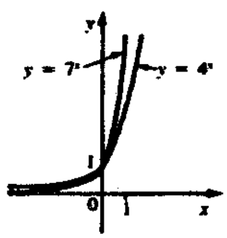
x	h(x)
-4	0.68
-2	0.83
0	1
2	1.21
4	1.46



2.

x	7^x	4^x
-2	$\frac{1}{49}$	$\frac{1}{16}$
-1	$\frac{1}{7}$	$\frac{1}{4}$
0	1	1
1	7	4
2	49	16

$y = 4^x$ and $y = 7^x$

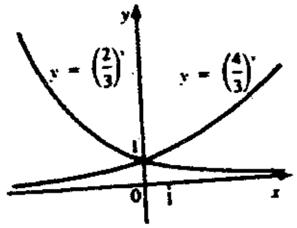


Exercise 8.1

3.

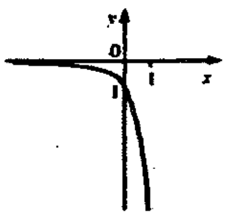
x	$\left(\frac{2}{3}\right)^x$	$\left(\frac{4}{3}\right)^x$
−4	5.06	0.32
−2	2.25	0.56
0	1	1
2	0.44	1.78
4	0.20	3.16

$y = \left(\frac{2}{3}\right)^x$ and $y = \left(\frac{4}{3}\right)^x$

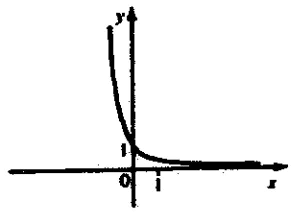


4. All functions have domain = the set of all real numbers (R: Range, HA: Horizontal Asymptote).

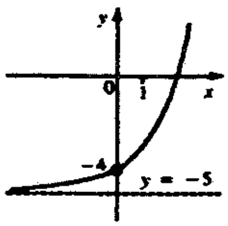
(a) $f(x) = -10^x$, $R = (-\infty, 0)$,
HA: $y = 0$



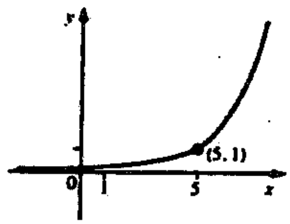
(b) $f(x) = 10^{-x}$, $R = (0, \infty)$,
HA: $y = 0$



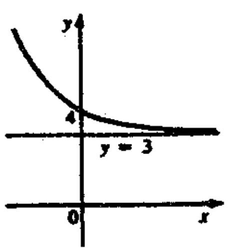
(c) $g(x) = 2^x - 5$, $R = (-5, \infty)$,
HA: $y = -5$



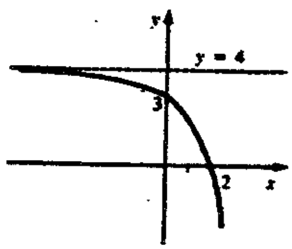
(d) $g(x) = 2^{x-5}$, $R = (0, \infty)$,
HA: $y = 0$



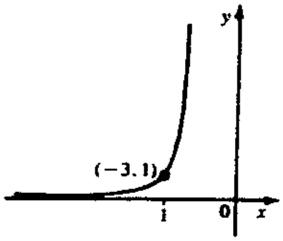
(e) $y = 3 + \left(\frac{1}{2}\right)^x$, $R = (3, \infty)$,
HA: $y = 3$



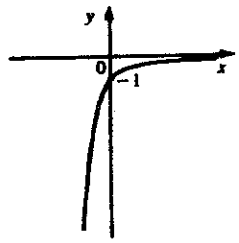
(f) $y = 4 - 2^x$, $R = (-\infty, 4)$,
HA: $y = 4$



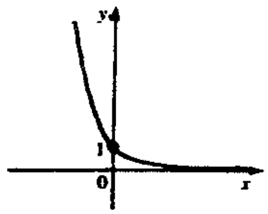
(g) $y = 10^{x+3}$, $R = (0, \infty)$,
HA: $y = 0$



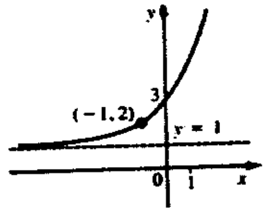
(h) $y = -\left(\frac{1}{10}\right)^x$, $R = (-\infty, 0)$,
HA: $y = 0$



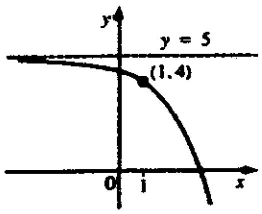
(i) $y = 2^{-2x}$, $R = (0, \infty)$,
HA: $y = 0$



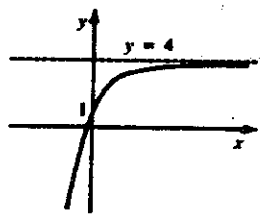
(j) $y = 1 + 2^{x+1}$, $R = (1, \infty)$,
HA: $y = 1$



(k) $y = 5 - 2^{x-1}$, $R = (-\infty, 5)$,
HA: $y = 5$



(l) $y = 1 + 3(1 - 10^{-x})$, $R = (-\infty, 4)$,
HA: $y = 4$



5. (a) $\lim_{x \to -\infty} 4^x = 0$, since $4 > 1$

(b) $\lim_{x \to \infty} (0.9)^x = 0$, since $0.9 < 1$

(c) $\lim_{x \to \infty} 10^{2x-1} = \infty$, since $2x - 1 \to \infty$ as $x \to \infty$, and $10 > 1$

(d) $\lim_{x \to \infty} 3^{-x} = 0$, since $-x \to -\infty$ as $x \to \infty$, and $3 > 1$

(e) $\lim_{x \to 0^+} 5^{\frac{1}{x}} = \infty$, since $\frac{1}{x} \to \infty$ as $x \to 0^+$, and $5 > 1$

(f) $\lim_{x \to 0^-} 5^{\frac{1}{x}} = 0$, since $\frac{1}{x} \to -\infty$ as $x \to 0^-$, and $5 > 1$

(g) $\lim_{x \to \infty} 10^{-x^2} = 0$, since $-x^2 \to -\infty$ as $x \to \infty$, and $10 > 1$

(h) $\lim\limits_{x \to \infty} 4^{\frac{1}{x}} = 1$, since $\frac{1}{x} \to 0$ as $x \to \infty$, and $4^0 = 1$

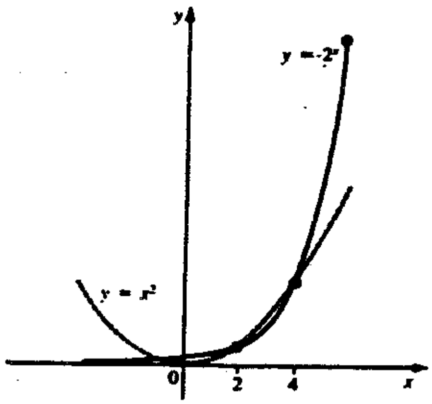
(i) $\lim\limits_{x \to -1^+} 8^{\frac{x}{x+1}} = 0$, since $\frac{x}{x+1} \to -\infty$ as $x \to -1^+$, and $8 > 1$

(j) $\lim\limits_{t \to 0^-} 2^{\csc t} = 0$, since $\csc t \to -\infty$ as $t \to 0^-$, and $2 > 1$

6. (a)

x	x^2	2^x
0	0	1
1	1	2
2	4	4
3	9	8
4	16	16
5	25	32
6	36	64
7	49	128
8	64	256
9	81	512
10	100	1024
15	225	32 768
20	400	1 048 576

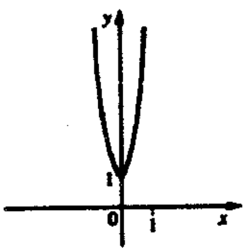
(b) $f(x) = x^2$ and $g(x) = 2^x$



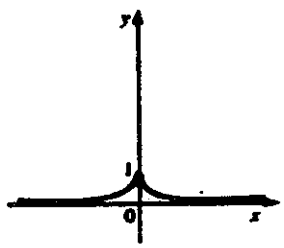
7. L.H.S. $= \dfrac{f(x+h) - f(x)}{h}$, but $f(x) = 10^x$ so L.H.S. $= \dfrac{10^{x+h} - 10^x}{h} = \dfrac{10^x 10^h - 10^x}{h}$

Factoring, we obtain L.H.S. $= 10^x\left(\dfrac{10^h - 1}{h}\right) =$ R.H.S.

8. (a) $y = 10^{|x|}$



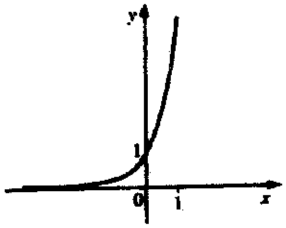
(b) $y = 10^{-|x|}$



EXERCISE 8.2

1. (a) $\frac{2}{e^{-x}} = 2e^{x}$ (b) $(e^{x})^{4} = e^{4x}$
   (c) $e^{1-x}e^{3x} = e^{2x+1}$ (d) $e^{x}e^{-x} = e^{0} = 1$
   (e) $e^{2x}(1 - 5e^{3x}) = e^{2x} - 5e^{5x}$ (f) $\frac{6e^{8x}}{e^{3x}} = 6e^{5x}$

2. (a) $y = 5^{x}$



(b)

h	$\frac{5^{h}-1}{h}$
0.1	1.746 189
0.01	1.622 459
0.001	1.610 734
0.0001	1.609 567

Slope of secant line.

   (c) $\lim_{h \to 0} \frac{5^{h}-1}{h} \doteq 1.61$
   (d) It is the slope of the tangent line to the curve $y = 5^{x}$ at the point (0,1).

3. (a) $\lim_{h \to 0} \frac{2.7^{h}-1}{h} \doteq 0.99$ (b) $\lim_{h \to 0} \frac{2.8^{h}-1}{h} \doteq 1.03$

4. (a) $y = 2e^{-x}$ so $y' = -2e^{-x}$
   (b) $y = x^{4}e^{x}$ so $y' = 4x^{3}e^{x} + x^{4}e^{x} = x^{3}e^{x}(4 + x)$
   (c) $y = e^{2x}\sin 3x$ so $y' = 2e^{2x}\sin 3x + 3e^{2x}\cos 3x = e^{2x}(2\sin 3x + 3\cos 3x)$
   (d) $y = e^{\sqrt{x}}$ so $y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (e) $y = e^{\tan x}$ so $y' = (\sec^{2}x)e^{\tan x}$
   (f) $y = \tan(e^{x})$ so $y' = (e^{x})\sec^{2}(e^{x})$ (g) $y = \frac{e^{x}}{x}$ so $y' = \frac{xe^{x} - e^{x}}{x^{2}} = \frac{e^{x}}{x^{2}}(x - 1)$
   (h) $y = \frac{e^{x}}{1 - e^{2x}}$ so $y' = \frac{e^{x}(1 - e^{2x}) + 2e^{2x}e^{x}}{(1 - e^{2x})^{2}} = \frac{e^{x}(1 + e^{2x})}{(1 - e^{2x})^{2}}$
   (i) $y = e^{\sin(x^{2})}$ so $y' = 2x\cos(x^{2})e^{\sin(x^{2})}$
   (j) $y = xe^{\cot 4x}$ so $y' = e^{\cot 4x} - 4x(\csc^{2}4x)e^{\cot 4x} = e^{\cot 4x}(1 - 4x\csc^{2}4x)$
   (k) $y = (1 + 5e^{-10x})^{4}$ so
   $y' = 4(1 + 5e^{-10x})^{3}(-50e^{-10x}) = -200e^{-10x}(1 + 5e^{-10x})^{3}$
   (l) $y = \sqrt{x + e^{1-x^{2}}}$ so $y' = \frac{1 - 2xe^{1-x^{2}}}{2\sqrt{x + e^{1-x^{2}}}}$

Exercise 8.2

5. $y = 1 + xe^{2x} \Rightarrow y'(x) = 2xe^{2x} + e^{2x} = e^{2x}(2x+1) \Rightarrow y'(0) = e^0(0+1) = 1$ and $y(0) = 1$, so the tangent is $y - 1 = 1(x - 0)$ which is $x - y + 1 = 0$.

6. $e^{xy} = 2x + y \Rightarrow e^{xy}\left[x\dfrac{dx}{dy} + y\right] = 2 + \dfrac{dx}{dy} \Rightarrow \dfrac{dy}{dx} = \dfrac{2 - ye^{xy}}{xe^{xy} - 1}$

7. $f(x) = e^{2x} \Rightarrow f'(x) = 2e^{2x} \Rightarrow f''(x) = 4e^{2x} \Rightarrow \cdots \Rightarrow f^{(6)}(x) = 64e^{2x} \Rightarrow$ $f^{(6)}(0) = 64$

8. $f(x) = x^2e^{-x} \Rightarrow f'(x) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2 - x)$. Note: $e^{-x} > 0$ for all x. So $f'(x) > 0$ when $x > 0$ and $x < 2$ (i.e., $0 < x < 2$) **or** $x < 0$ and $x > 2$ (empty set), so f increases on $(0,2)$. f decreases on $(-\infty,0)$ and $(2,\infty)$.

9. $f(x) = \dfrac{e^x}{x},\ x > 0 \Rightarrow f'(x) = \dfrac{xe^x - e^x}{x^2} = \dfrac{e^x(x-1)}{x^2} > 0$ when $x > 1$.
f increases on $(1,\infty)$; f decreases on $(0,1)$; thus absolute minimum $= f(1) = e$.

10. $f(x) = xe^x \Rightarrow f'(x) = e^x + xe^x = e^x(1+x) > 0$ when $x > -1$.

(a) f increases on $(-1,\infty)$; f decreases on $(-\infty,-1)$; thus the absolute minimum $= f(-1) = -\frac{1}{e}$

(b) $f''(x) = e^x + e^x(1+x) = e^x(2+x)$
CU: $f''(x) > 0$ on $(-2,\infty)$; CD: $f''(x) < 0$ on $(-\infty,-2)$

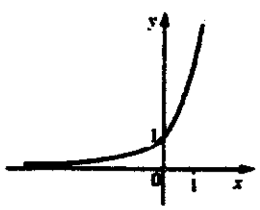
(c) $f(-2) = -\dfrac{2}{e^2}$ so the inflection point is $(-2, -\dfrac{2}{e^2})$

11. (a) $\lim\limits_{x\to\infty} e^{-x} = 0$, since $-x \to -\infty$ as $x \to \infty$, and $e > 0$

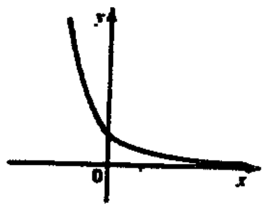
(b) $\lim\limits_{x\to-\infty} e^{-x} = \infty$, since $-x \to \infty$ as $x \to -\infty$, and $e > 0$

(c) $\lim\limits_{t\to\frac{\pi}{2}^+} e^{\tan t} = 0$, since $\tan t \to -\infty$ as $t \to \frac{\pi}{2}^+$, and $e > 0$

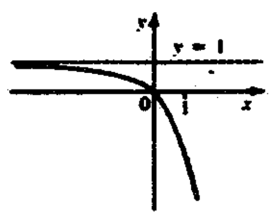
12. (a) $y = e^x$



(b) (i) $y = e^{-x}$



(ii) $y = 1 - e^x$



Exercise 8.2

13. (a) $\boxed{y = x e^{x^2}}$ A. Domain: R; B. Intercepts: both 0

C. Symmetry: $f(-x) = -f(x) \Rightarrow$ the function is odd, symmetric about the origin

D. Asymptotes: No V.A., $\lim_{x \to \infty} x e^{x^2} = \infty$ and $\lim_{x \to -\infty} x e^{x^2} = -\infty \Rightarrow$ No H.A.

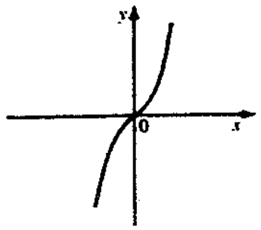
E. Inc/Dec: $y' = e^{x^2}(1 + 2x^2) \Rightarrow f$ increases on $(-\infty, \infty)$

F. No Maxima or Minima;

G. Concavity: $y'' = 2x e^{x^2}(3 + 2x^2) \Rightarrow$ CU on $(0, \infty)$, CD on $(-\infty, 0)$;

Inflection point: $(0,0)$.

H.



(b) $\boxed{y = e^{\frac{1}{x^2}}}$ A. Domain: $(-\infty, 0) \cup (0, \infty)$ B. No Intercepts

C. Symmetry: $f(-x) = f(x) \Rightarrow$ the function is even, symmetric about the y-axis

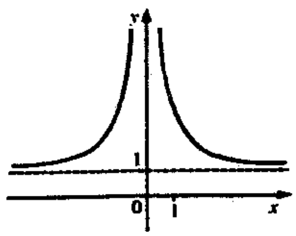
D. Asymptotes: $\lim_{x \to 0} e^{\frac{1}{x^2}} = \infty \Rightarrow x = 0$ is V.A., $\lim_{x \to \pm\infty} e^{\frac{1}{x^2}} = e^0 = 1 \Rightarrow y = 1$ is H.A.

E. Inc/Dec: $y' = \dfrac{-2e^{\frac{1}{x^2}}}{x^3} \Rightarrow f$ increases on $(-\infty, 0)$, f decreases on $(0, \infty)$

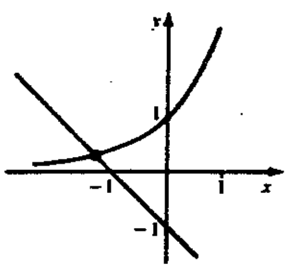
F. No Maxima or Minima

G. Concavity: $y'' = \dfrac{-2e^{\frac{1}{x^2}}}{x^6}(3x^2 + 2) \Rightarrow$ CU on $(-\infty, 0) \cup (0, \infty)$; No Inflection point.

H.



14. (a) $y = -x - 1$ and $y = e^x$



(b) $e^x + x + 1 = 0$; If $f(x) = e^x + x + 1$,

$f'(x) = e^x + 1$ and $x_{n+1} = x_n - \dfrac{e^{x_n} + x_n + 1}{e^{x_n} + 1}$

$x_0 = -1$

$x_1 \doteq -1.268\ 941$

$x_2 \doteq -1.278\ 455$

$x_4 \doteq -1.278\ 465$ $\quad x_5 \doteq -1.278\ 465$

15. (a) $\boxed{y = e^{-\frac{1}{x}}}$ A. Domain: $(-\infty, 0) \cup (0, \infty)$

B. No Intercepts; C. No Symmetry

D. Asymptotes: $\lim_{x \to 0^-} e^{-\frac{1}{x}} = \infty$ and $\lim_{x \to 0^+} e^{-\frac{1}{x}} = 0 \Rightarrow x = 0$ is V.A., $\lim_{x \to \pm\infty} e^{-\frac{1}{x}} = 1 \Rightarrow$ $y = 1$ is H.A.;

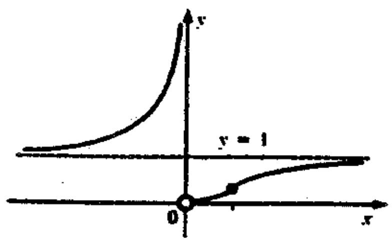
E. Inc/Dec: $y' = \frac{e^{-\frac{1}{x}}}{x^2} > 0 \Rightarrow$ f increases on $(-\infty, 0)$ and $(0, \infty)$.

F. No Maxima or Minima

G. Concavity: $y'' = \frac{e^{-\frac{1}{x}}}{x^4}(1 - 2x) \Rightarrow$ CU on $(-\infty, 0)$ and $(0, \frac{1}{2})$, CD on $(\frac{1}{2}, \infty)$

Inflection point: $\left(\frac{1}{2}, \frac{1}{e^2}\right)$.

H.

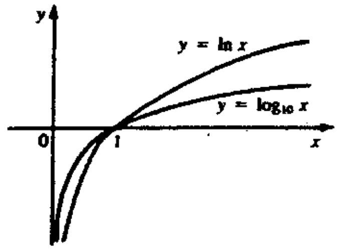


16. $f(x) = xe^{-x}$

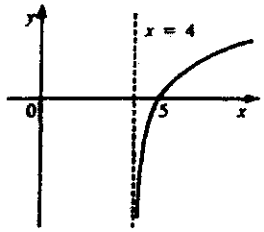
$$\begin{aligned}
&\Rightarrow f'(x) = e^{-x} - xe^{-x} && = e^{-x}(1 - x)\\
&\Rightarrow f''(x) = -e^{-x} - (e^{-x} - xe^{-x}) && = e^{-x}(x - 2)\\
&\Rightarrow f'''(x) = 2e^{-x} + (e^{-x} - xe^{-x}) && = e^{-x}(3 - x)\\
&\Rightarrow f^{(4)}(x) = -3e^{-x} - (e^{-x} - xe^{-x}) && = e^{-x}(x - 4)\\
&\vdots && \vdots\\
&\Rightarrow f^{(1\,000\,000)}(x) = e^{-x}(x - 1\,000\,000)
\end{aligned}$$

EXERCISE 8.3

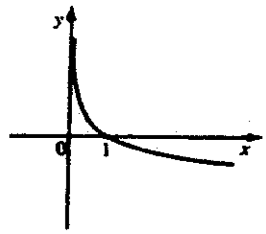
1. $y = \ln x$ and $y = \log_{10} x$



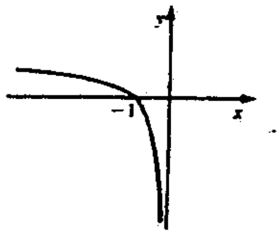
2. (a) $f(x) = \log_2(x-4)$; Domain: $(4, \infty)$
Range: R, Asymptote: $x = 4$



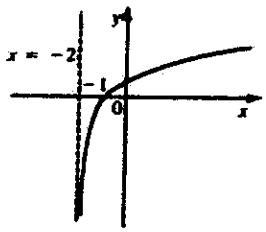
(b) $f(x) = -\log_{10} x$; Domain: $(0, \infty)$
Range: R, Asymptote: $x = 0$



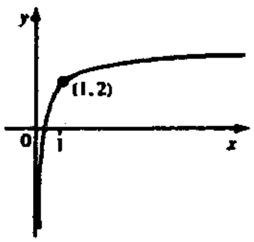
(c) $g(x) = \log(-x)$; Domain: $(-\infty, 0)$
Range: R, Asymptote: $x = 0$



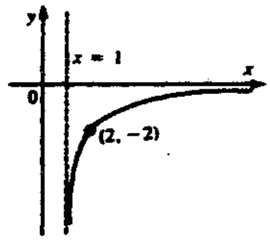
(d) $g(x) = \ln(x+2)$; Domain: $(-2, \infty)$
Range: R, Asymptote: $x = -2$



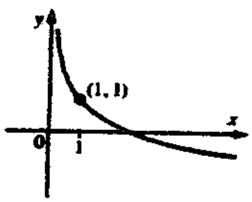
(e) $y = 2 + \log_2 x$; Domain: $(0, \infty)$
Range: R, Asymptote: $x = 0$



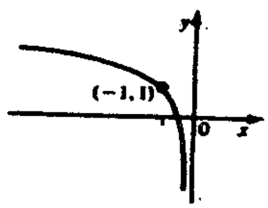
(f) $y = \log_2(x-1) - 2$; Domain: $(1, \infty)$
Range: R, Asymptote: $x = 1$



(g) $y = 1 - \ln x$; Domain: $(0, \infty)$
Range: R, Asymptote: $x = 0$

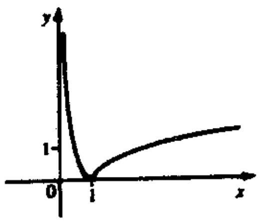


(h) $y = 1 + \ln(-x)$; Domain: $(-\infty, 0)$
Range: R, Asymptote: $x = 0$



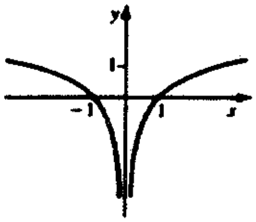
(i) $y = |\ln x|$; Domain: $(0, \infty)$

Range: $[0, \infty)$ Asymptote: $x = 0$



(j) $y = \ln |x|$; Domain: $(-\infty, 0) \cup (0, \infty)$

Range: R, Asymptote: $x = 0$



3. (a) $e^{\ln 5} = 5$

(b) $\ln e^2 = 2$

(c) $2 \ln e = 2$

(d) $e^{5 \ln 2} = 2^5 = 32$

(e) $\ln \sqrt{e} = \frac{1}{2}$

(f) $\ln 2 + 2 \ln 3 - \ln 18 = \ln \frac{(2)3^2}{18} = \ln 1 = 0$

4. (a) $e^x = 4$, so $x = \ln 4$

(b) $\ln x = 6$, so $x = e^6$

(c) $\ln(2x - 1) = 1$, so $2x - 1 = e$, $x = \frac{1}{2}(e + 1)$

(d) $e^{3x+5} = 10$, so $3x + 5 = \ln 10$, $x = \frac{1}{3}(\ln 10 - 5)$

(e) $\ln(e^{3-x}) = 8$, so $3 - x = 8$, $x = -5$

(f) $\ln x = \ln 4 + \ln 7$, so $\ln x = \ln 28$, $x = 28$

(g) $\ln(\ln x) = 2$, so $\ln x = e^2$, $x = e^{e^2}$

(h) $e^{e^x} = 5$, so $e^x = \ln 5$, $x = \ln(\ln 5)$

5. (a) $\ln(x + 1) = 3$, so $x + 1 = e^3$, $x = e^3 - 1 \doteq 19.085\ 537$

(b) $e^{-x} = \frac{1}{2}$, so $-x = \ln \frac{1}{2}$, $x = -\ln \frac{1}{2} \doteq 0.693\ 147$

(c) $e^{5x+3} = 10$, so $5x + 3 = \ln 10$, $x = \frac{1}{5}(\ln 10 - 3) \doteq -0.139\ 483$

(d) $2^{x-5} = 3$, so $x - 5 = \log_2 3$, $x = \log_2 3 + 5 = \frac{\ln 3}{\ln 2} + 5 \doteq 6.584\ 963$

6. (a) $\frac{1}{3}\ln x + 2\ln(3x - 5) = \ln \sqrt[3]{x} + \ln(3x - 5)^2 = \ln\left[\sqrt[3]{x}(3x - 5)^2\right]$

(b) $2\ln x - \frac{1}{2}\ln(x^2 - 1) + 3\ln(x^2 + 1) = \ln x^2 - \ln\sqrt{x^2 - 1} + \ln(x^2 + 1)^3$

$= \ln\left(\frac{x^2(x^2 + 1)^3}{\sqrt{x^2 - 1}}\right)$

7. (a) $f(x) = \log_{10}(2 + 5x)$: $2 + 5x > 0$, Domain $= (-\frac{2}{5}, \infty)$

(b) $f(x) = \log_2(10 - 3x)$: $10 - 3x > 0$, Domain $= (-\infty, \frac{10}{3})$

(c) $g(x) = \log_3(x^2 - 1)$: $x^2 - 1 > 0$, Domain $= (-\infty, -1) \cup (1, \infty)$

(d) $g(x) = \ln(x - x^2)$: $(x > 0) \cap (1 - x > 0)$ or $(x < 0) \cap (1 - x < 0)$, Domain $= (0, 1)$

(e) $h(x) = \ln x + \ln(2 - x)$: $(x > 0) \cap (2 - x > 0)$, Domain $= (0, 2)$

(f) $h(x) = \sqrt{x-2} - \ln(10-x)$: $(x-2 \geq 0) \cap (10-x > 0)$, Domain $= [2, 10)$

8. $f(x) = \ln x^2$: $x^2 > 0$, $|x| > 0$, Domain $= (-\infty, 0) \cup (0, \infty)$

$g(x) = 2\ln x$: $x > 0$, Domain $= (0, \infty)$

9. (a) $\lim_{x \to -4^+} \ln(x+4) = -\infty$, since $(x+4) \to 0^+$ as $x \to -4^+$

(b) $\lim_{x \to \infty} \ln(x+4) = \infty$, since $(x+4) \to \infty$ as $x \to \infty$

(c) $\lim_{x \to 1^+} \log_{10}(x^2 - x) = -\infty$, since $(x^2 - x) \to 0^+$ as $x \to 1^+$

(d) $\lim_{t \to \pi^-} \ln(\sin t) = -\infty$, since $\sin t \to 0^+$ as $t \to \pi^-$

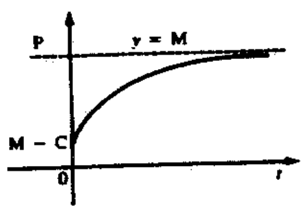
10. (a) $\log_2 7 = \frac{\ln 7}{\ln 2} \doteq 2.807\ 355$ (b) $\log_5 2 = \frac{\ln 2}{\ln 5} \doteq 0.430\ 677$

(c) $\log_3 11 = \frac{\ln 11}{\ln 3} \doteq 2.182\ 658$ (d) $\log_6 92 = \frac{\ln 92}{\ln 6} \doteq 2.523\ 658$

11. L.H.S $= \log e = \log_{10} e = \frac{\ln e}{\ln 10} = \frac{1}{\ln 10} =$ R.H.S.

12. $(\log_2 5)(\log_5 7) = \frac{\ln 5}{\ln 2} \times \frac{\ln 7}{\ln 5} = \frac{\ln 7}{\ln 2} = \log_2 7$

13. (a) $P(t) = M - Ce^{-kt}$



(b) Isolate t in $P = M - Ce^{-kt}$:

$$Ce^{-kt} = M - P$$

$$e^{-kt} = \left(\frac{M-P}{C}\right)$$

$$-kt = \ln\left(\frac{M-P}{C}\right)$$

$$\text{So } t = -\frac{1}{k}\ln\left(\frac{M-P}{C}\right)$$

14. Let $y = \log_4 17$ and $x = \log_5 24$. Therefore $4^y = 17$ and $5^x = 24$. But $4^2 < 17$, so $y > 2$, and $5^2 > 24$, so $x < 2$. We have $x < 2 < y$, so $x < y$, and $\log_5 24 < \log_4 17$.

Exercise 8.3

15. (a) $f(x) = \log_2(\log_{10} x)$: Domain of $\log_{10} x = (0, \infty)$, so Domain of

$\log_2(\log_{10} x) = (1, \infty)$

(b) Inverse of $y = \log_2(\log_{10} x)$ is $x = \log_2(\log_{10} y) \Rightarrow 2^x = \log_{10} y \Rightarrow 10^{2^x} = y$,

so the inverse function is $f^{-1}(x) = 10^{2^x}$.

16. (a) $\ln(\ln(\ln x))$: Domain of $\ln x$ is $(0, \infty) \Rightarrow$ Domain of $\ln(\ln x)$ is $(1, \infty) \Rightarrow$ Domain of $\ln(\ln(\ln x))$ is (e, ∞).

(b) Inverse of $y = \ln(\ln(\ln x))$ is $x = \ln(\ln(\ln y)) \Rightarrow e^x = \ln(\ln y) \Rightarrow e^{e^x} = \ln y \Rightarrow$
$e^{e^{e^x}} = y$, so the inverse function is $f^{-1}(x) = e^{e^{e^x}}$.

17. $4^x - 2^{x+1} = 3$, so $2^{2x} - 2^{x+1} - 3 = 0$, $\left(2^x + 1\right)\left(2^x - 3\right) = 0$,
$2^x = -1$ (not possible) OR $2^x = 3$, $x = \log_2 3 = \frac{\ln 3}{\ln 2} \doteq 1.584\ 963$

18. $\log_2 x + \log_4 x + \log_8 x = 11 \Rightarrow \log_2 x + \frac{\log_2 x}{\log_2 4} + \frac{\log_2 x}{\log_2 8} = 11 \Rightarrow$

$\log_2 x\left[1 + \frac{1}{\log_2 4} + \frac{1}{\log_2 8}\right] = 11 \Rightarrow \log_2 x\left[1 + \frac{1}{2} + \frac{1}{3}\right] = 11 \Rightarrow$

$\log_2 x\left[\frac{11}{6}\right] = 11 \Rightarrow \log_2 x = 11\left[\frac{6}{11}\right] \Rightarrow \log_2 x = 6 \Rightarrow x = 2^6 = 64$

EXERCISE 8.4

1. (a) $f(x)=x^2\ln x \Rightarrow f'(x)=2x\ln x+x=x(2\ln x+1)$

   (b) $f(x)=\sqrt{\ln x} \Rightarrow f'(x)=\dfrac{1}{2x\sqrt{\ln x}}$ (c) $g(x)=\ln(x^3+1) \Rightarrow g'(x)=\dfrac{3x^2}{x^3+1}$

   (d) $g(x)=\ln(5x) \Rightarrow g'(x)=\frac{1}{x}$ (e) $y=\sin(\ln x) \Rightarrow y'=\frac{1}{x}\cos(\ln x)$

   (f) $y=\ln(\sin x) \Rightarrow y'=\dfrac{\cos x}{\sin x}=\cot x$

   (g) $y=\dfrac{\ln x}{x^3} \Rightarrow y'=\dfrac{\frac{1}{x}(x^3)-3x^2(\ln x)}{x^6}=\dfrac{1-3\ln x}{x^4}$

   (h) $y=(x+\ln x)^3 \Rightarrow y'=3(x+\ln x)^2(1+\frac{1}{x})$

   (i) $y=\ln|2x+1| \Rightarrow y'=\dfrac{2}{2x+1}$

   (j) $y=\ln\left[\dfrac{x+1}{x-1}\right] \Rightarrow y'=\dfrac{x-1}{x+1}\times\dfrac{x-1-x-1}{(x-1)^2}=-\dfrac{2}{x^2-1}$

   (k) $y=\ln\sqrt{\dfrac{x}{2x+3}} = \frac{1}{2}[\ln x-\ln(2x+3)] \Rightarrow$

   $y'=\frac{1}{2}\left[\frac{1}{x}-\dfrac{2}{2x+3}\right]=\frac{1}{2}\left[\dfrac{2x+3-2x}{x(2x+3)}\right]=\dfrac{3}{2x(2x+3)}$

   (l) $y=\ln\dfrac{x}{\sqrt{x^2+1}} = \ln x-\frac{1}{2}\ln(x^2+1) \Rightarrow$

   $y'=\frac{1}{x}-\frac{1}{2}\left(\dfrac{2x}{x^2+1}\right)=\frac{1}{x}-\dfrac{x}{x^2+1}=\dfrac{1}{x(x^2+1)}$

   (m) $y=\ln(\sec x+\tan x) \Rightarrow y'=\dfrac{\sec x\tan x+\sec^2 x}{\sec x+\tan x}=\sec x$

   (n) $y=\tan[\ln(1-3x)] \Rightarrow y'=\dfrac{-3\sec^2[\ln(1-3x)]}{1-3x}$

2. (a) $f(x)=\ln(\ln x) \Rightarrow f'(x)=\dfrac{1}{\ln x}\times\dfrac{1}{x}=\dfrac{1}{x\ln x}$

   (b) Domain of $f=(1,\infty)$; Domain of $\dfrac{1}{x\ln x}=(0,1)\cup(1,\infty)$.
   but Domain of $f'\subseteq$ Domain of f so Domain of $f'=(1,\infty)$

3. (a) $f(x)=\log_2(x^2+1) \Rightarrow f'(x)=\dfrac{2x}{(x^2+1)\ln 2}$

   (b) $g(x)=x\log_{10}x \Rightarrow g'(x)=\log_{10}x+\dfrac{1}{\ln 10}$

   (c) $F(x)=\log_5(3x-8) \Rightarrow F'(x)=\dfrac{3}{(3x-8)\ln 5}$

   (d) $G(x)=\dfrac{1+\log_3 x}{x} \Rightarrow$

$$G'(x) = \frac{\frac{x}{x \ln 3} - 1 - \log_3 x}{x^2} = \frac{1 - \ln 3 - (\ln 3)\log_3 x}{(\ln 3)x^2} = \frac{1 - \ln x - \ln 3}{(\ln 3)x^2}$$

4. (a) $y = x^3 + 3^x \Rightarrow y' = 3x^2 + 3^x \ln 3$

(b) $y = 2^{x^4 - x} \Rightarrow y' = (4x^3 - 1)(\ln 2)2^{x^4 - x}$

(c) $y = x5^{\sqrt{x}} \Rightarrow y' = \frac{x \ln 5}{2\sqrt{x}}5^{\sqrt{x}} + 5^{\sqrt{x}} = \frac{5^{\sqrt{x}}}{2}(\sqrt{x}\ln 5 + 2)$

(d) $y = 10^{\tan \pi x} \Rightarrow y' = \pi\left[\sec^2 \pi x\right]\ln 10\left[10^{\tan \pi x}\right]$

5. (a) $y = \ln(x-1)$, $(2,0) \Rightarrow y' = \frac{1}{x-1}$ so $y'(2) = 1$; the tangent is $y - 0 = 1(x-2)$ or $x - y - 2 = 0$.

(b) $y = x^2 \ln x$, $(1,0) \Rightarrow y' = 2x\ln x + x$ so $y'(1) = 1$; the tangent is $y - 0 = 1(x-1)$ or $x - y - 1 = 0$.

(c) $y = 10^x$, $(1,10) \Rightarrow y' = 10^x \ln 10$ so $y'(1) = 10(\ln 10)$; the tangent is $y - 10 = 10 \ln 10 (x-1)$ or $10(\ln 10)x - y - 10(\ln 10 - 1) = 0$.

(d) $y = \log_{10} x$, $(100,2) \Rightarrow y' = \frac{1}{x \ln 10}$ so $y'(100) = \frac{1}{100(\ln 10)}$; the tangent is $y - 2 = \frac{1}{100(\ln 10)}(x - 100)$ or $x - 100(\ln 10)y + 100(2\ln 10 - 1) = 0$.

6. $\ln(x+y) = y - 1 \Rightarrow \frac{1+y'}{x+y} = y' \Rightarrow (x+y-1)y' = 1 \Rightarrow y' = \frac{1}{x+y-1}$

7. (a) $f(x) = x\ln x \Rightarrow f'(x) = \ln x + 1$ so f increases on $(\frac{1}{e}, \infty)$ and f decreases on $(0, \frac{1}{e})$, so $f(\frac{1}{e}) = -\frac{1}{e}$ is the absolute minimum.

(b) $f''(x) = \frac{1}{x}$ so f is concave upward on $(0, \infty)$.

8. (a) $f(x) = x(\ln x)^2 \Rightarrow f'(x) = (\ln x)^2 + 2x(\ln x)\frac{1}{x} = (\ln x)\left[\ln x + 2\right]$ so f increases on $(0, e^{-2}) \cup (1, \infty)$ and f decreases on $(e^{-2}, 1)$. Thus, $f(e^{-2}) = \frac{4}{e^2}$ is a local maximum and $f(1) = 0$ is a local minimum.

(b) $f''(x) = \frac{2}{x} + 2(\ln x)\frac{1}{x} = \frac{2}{x}(\ln x + 1)$ so f is concave upward on (e^{-1}, ∞) and f is concave downward on $(0, e^{-1})$. Thus, $(\frac{1}{e}, \frac{1}{e})$ is the inflection point.

9. (a) $\boxed{y = \ln(9 - x^2)}$ A. Domain: $(-3, 3)$

B. Intercepts: y-intercept $= \ln 9$, x-intercepts $= \pm 2\sqrt{2}$

C. Symmetry: $f(-x) = f(x) \Rightarrow$ the function is even, symmetric about the y-axis

D. Asymptotes: $\lim_{x \to -3^+} \ln(9 - x^2) = -\infty$ and $\lim_{x \to 3^-} \ln(9 - x^2) = -\infty \Rightarrow x = -3$ and $x = 3$ are V.A., No H.A.

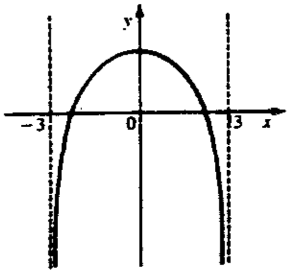
E. Inc/Dec: $y' = \frac{-2x}{9-x^2} \Rightarrow$ f increases on $(-3,0)$, f decreases on $(0,3)$

F. $f(0) = \ln 9$ is an absolute maximum

G. Concavity: $y'' = \frac{-18+2x^2+2x(-2x)}{(9-x^2)^2} = \frac{-2(x^2+9)}{(9-x^2)^2} \Rightarrow$ CD on $(-3,3)$;

No Inflection point.

H.



(b) $\boxed{y = x + \ln x}$ A. Domain: $(0,\infty)$

B. Intercepts: No y-intercept, (x-intercept $\doteq 0.567\,143$ by Newton's method)

C. No Symmetry

D. Asymptotes: $\lim_{x \to 0^+} (x + \ln x) = -\infty \Rightarrow x = 0$ is V.A., $\lim_{x\to\infty} (x + \ln x) = \infty$

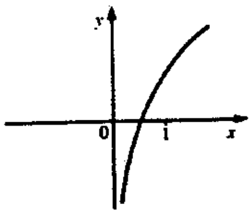
$\Rightarrow$ No H.A.

E. Inc/Dec: $y' = 1 + \frac{1}{x} \Rightarrow$ f increases on $(0,\infty)$

F. No maxima or minima

G. Concavity: $y'' = -x^{-2} < 0 \Rightarrow$ CD on $(0,\infty)$; No Inflection point.

H.



(c) $\boxed{y = (\ln x)^2}$ A. Domain: $(0,\infty)$

B. Intercepts: No y-intercept, x-intercept = 1

C. No Symmetry

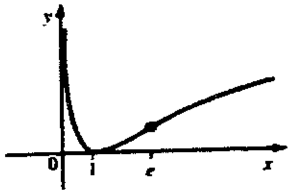
D. Asymptotes: $\lim_{x \to 0^+} (\ln x)^2 = \infty \Rightarrow x = 0$ is V.A., $\lim_{x\to\infty} (\ln x)^2 = \infty \Rightarrow$ No H.A.

E. Inc/Dec: $y' = \frac{2}{x}\ln x \Rightarrow$ f increases on $(1,\infty)$, f decreases on $(0,1)$

F. $f(1) = 0$ is an absolute minimum

G. Concavity: $y'' = -2x^{-2}\ln x + 2x^{-2} = \frac{2}{x^2}(1 - \ln x) \Rightarrow$ CU on $(0,e)$, CD on (e,∞); $(e,1)$ is the Inflection point.

H.



(d) $\boxed{y = \ln(\cos x)}$ A. Domain: $\{x \mid \frac{(4n-1)\pi}{2} < x < \frac{(4n+1)\pi}{2}$, where $n \in I\}$

B. Intercepts: y-intercept $= 0$, x-intercepts $= 2n\pi$, $n \in I$

C. Symmetry: $f(-x) = f(x) \Rightarrow$ the function is even, symmetric about the y-axis: period 2π

D. Asymptotes: $\lim_{x \to \frac{(4n-1)\pi}{2}^+} \ln(\cos x) = -\infty$, and $\lim_{x \to \frac{(4n+1)\pi}{2}^-} \ln(\cos x) = -\infty \Rightarrow$

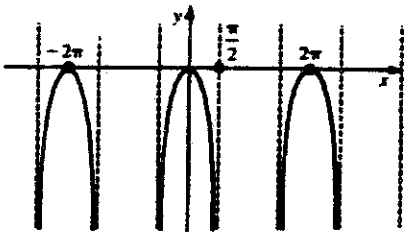
$x = \frac{(4n-1)\pi}{2}$ and $x = \frac{(4n+1)\pi}{2}$, $n \in I$ are V.A., No H.A.

E. Inc/Dec: $y' = \frac{-\sin x}{\cos x} = -\tan x \Rightarrow$ f increases on $\left(\frac{(4n-1)\pi}{2}, 2n\pi\right]$, f decreases on each interval $\left[2n\pi, \frac{(4n+1)\pi}{2}\right)$, $n \in I$.

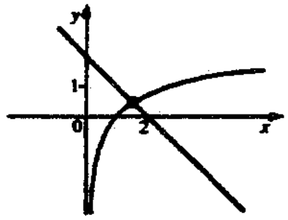
F. $f(2n\pi) = 0$, $n \in I$ are maxima

G. Concavity: $y'' = -\sec^2 x \Rightarrow$ f is CD on $\left(\frac{(4n-1)\pi}{2}, \frac{(4n+1)\pi}{2}\right)$;
No Inflection points.

H.



10. (a) $y = 2 - x$ and $y = \ln x$



(b) $\ln x + x - 2 = 0$; If $f(x) = \ln x + x - 2$, $f'(x) = \frac{1}{x} + 1$ and

$$x_{n+1} = x_n - \frac{\ln x_n + x_n - 2}{\frac{1}{x_n} + 1}$$

$x_1 = 1.5$

$x_2 \doteq 1.556\ 721$

$x_3 \doteq 1.557\ 146$ $\quad x_4 \doteq 1.557\ 146$

Exercise 8.4

11. We start with: $\log_b x = \frac{\ln x}{\ln b}$ (change of base formula) and take the derivative of both sides, using $\frac{d}{dx}\ln x = \frac{1}{x}$ (formula 1). This gives:

$\frac{d}{dx}\log_b x = \frac{\frac{1}{x}}{\ln b} \Rightarrow \frac{d}{dx}\log_b x = \frac{1}{x\ln b}$ (formula 4).

12. $\lim_{x \to 0}(1+3x)^{\frac{1}{x}}$. Let $t = 3x \Rightarrow \lim_{\frac{t}{3} \to 0}(1+t)^{\frac{3}{t}}$ but $t \to 0$ as $\frac{t}{3} \to 0$ so $\lim_{t \to 0}(1+t)^{\frac{3}{t}}$

$= \left[\lim_{t \to 0}(1+t)^{\frac{1}{t}}\right]^3 = e^3$

EXERCISE 8.5

1. $f(0) = y_0 = 1000$, $f(2) = 10\ 000$

   (a) $f(t) = y_0 e^{kt} \Rightarrow$ at $t = 2$, $10\ 000 = 1000e^{2k} \Rightarrow 10 = e^{2k} \Rightarrow k = \frac{1}{2}\ln 10$

   Thus, $f(t) = 1000\,e^{(\frac{1}{2}\ln 10)t} \Rightarrow f(t) = (1000)10^{\frac{t}{2}}$ after t hours.

   (b) $f(5) = (1000)10^{\frac{5}{2}} \doteq 316\ 228$ bacteria after 5 h.

   (c) $f'(t) = (1000)e^{(\frac{1}{2}\ln 10)t}(\frac{1}{2}\ln 10) \Rightarrow f'(5) = 500(\ln 10)e^{(\frac{5}{2}\ln 10)} \doteq 364\ 071$

   bacteria/h after 5h.

   (d) $15\ 000 = (1000)e^{(\frac{1}{2}\ln 10)t} \Rightarrow \ln 15 = (\frac{1}{2}\ln 10)t \Rightarrow t = \frac{2\ln 15}{\ln 10} \doteq 2.35$ h, or about

   2 h 21 min when the population reaches 15 000.

2. $f(0) = y_0 = 400$, $f(1) = 1200$

   (a) $f(t) = y_0 e^{kt} \Rightarrow$ at $t = 1$, $1200 = 400e^{k} \Rightarrow 3 = e^{k} \Rightarrow k = \ln 3$

   Thus, $f(t) = 400\,e^{(\ln 3)t} = (400)3^{t}$ after t hours.

   (b) $2 = e^{(\ln 3)t} \Rightarrow \ln 2 = (\ln 3)t \Rightarrow t = \frac{\ln 2}{\ln 3} \doteq 0.63$ h, or about 37.5 min when the population doubles.

3. $f(0) = y_0 = 500$, doubling time $= 20$ min $= \frac{1}{3}$ h.

   (a) $2 = e^{\frac{1}{3}k} \Rightarrow \ln 2 = \frac{1}{3}k \Rightarrow k = 3\ln 2 \Rightarrow f(t) = (500)e^{(3\ln 2)t} = (500)2^{3t}$ after t hours.

   (b) $f(8) = (500)2^{24} \doteq 8.389 \times 10^{9}$ bacteria after 8 h.

   (c) $6000 = (500)2^{3t} \Rightarrow 12 = 2^{3t} \Rightarrow t = \frac{1}{3}\log_2 12 = \frac{\ln 12}{3\ln 2} \doteq 1.19$h, or about

   1 h 11 min for the population to reach 6000 bacteria.

4. $f(\frac{1}{4}) = 5000$, $f(1) = 40\ 000$

   (a) Population increases by a factor of $\frac{40\ 000}{5000} = 8$, in $1 - \frac{1}{4} = \frac{3}{4}$h. If it has doubled three times in $\frac{3}{4}$ h, its doubling time must be $\frac{1}{4}$ h. Since $f(\frac{1}{4}) = 5000$, $f(0)$, the initial size of the culture, must be equal to 2500.

   (b) $f(t) = y_0 e^{kt} \Rightarrow$ at $t = 1$, $40\ 000 = 2500e^{k} \Rightarrow 16 = e^{k} \Rightarrow k = \ln 16 = 4\ln 2$

   Thus, $f(t) = 2500\,e^{(4\ln 2)t} = (2500)16^{t}$ after t hours.

   (c) $f'(t) = 2500\,e^{(4\ln 2)t}(4\ln 2) \Rightarrow f'(\frac{1}{4}) = 2500\,e^{(4(\frac{1}{4})\ln 2)}(4\ln 2) \doteq 13\ 863$

   bacteria/h after 15 min.

   (d) $150\ 000 = (2500)e^{(4\ln 2)t} \Rightarrow 60 = e^{(4\ln 2)t} \Rightarrow \ln 60 = (4\ln 2)t \Rightarrow t = \frac{\ln 60}{4\ln 2}$

   $\doteq 1.48$ h, or about 1 h 28 min for the population to reach 150 000 bacteria.

5. rate of growth = 4% per annum, $f(1980) = y_0 = 275\ 000 \Rightarrow f(t) = y_0(1.04)^t \Rightarrow$
$f(t) = 275\ 000(1.04)^t$
   (a) $f(5) = 275\ 000(1.04)^5 \doteq 334\ 580$ people in 1985.
   (b) $f(20) = 275\ 000(1.04)^{20} \doteq 602\ 559$ people in 2000.

6. $f(1987) = y_0 = 5 \times 10^9$, doubling time = 35a. $\Rightarrow 2 = e^{35k} \Rightarrow \ln 2 = 35k \Rightarrow$
$k = \frac{\ln 2}{35} \Rightarrow f(t) = (5 \times 10^9)e^{(\frac{1}{35}\ln 2)t} = (5 \times 10^9)2^{\frac{1}{35}t}$
   (a) (i) $f(14) = (5 \times 10^9)2^{\frac{14}{35}} \doteq 6.6 \times 10^9$ people in 2001.
   (ii) $f(113) = (5 \times 10^9)2^{\frac{113}{35}} \doteq 4.7 \times 10^{10}$ people in 2100.
   (b) $2^{\frac{1}{35}t} = \frac{5 \times 10^{10}}{5 \times 10^9} = 10 \Rightarrow t = 35\frac{\ln 10}{\ln 2} \doteq 116$ a. The population will reach 50 billion in the year 2103.

7. U-238 has half-life of 4.5×10^9 years
   (a) $f(0) = y_0 = 100$; $\frac{1}{2} = e^{(4.5 \times 10^9)k} \Rightarrow k = \frac{-\ln 2}{4.5 \times 10^9}$
   $\Rightarrow f(t) = (100)e^{\left(\frac{-\ln 2}{4.5 \times 10^9}\right)t} \Rightarrow f(t) = (100)\ 2^{\left(\frac{-t}{4.5 \times 10^9}\right)}$ after t years.
   (b) $f(10\ 000) = (100)\ 2^{\left(-\frac{1 \times 10^4}{4.5 \times 10^9}\right)} \doteq 99.999\ 846$ mg after 10 000 a.
   (c) $f'(t) = (100)e^{\left(\frac{-\ln 2}{4.5 \times 10^9}\right)t}\left(\frac{-\ln 2}{4.5 \times 10^9}\right) \Rightarrow$
   $f'(10\ 000) = (100)e^{\left(\frac{-10\ 000\ln 2}{4.5 \times 10^9}\right)}\left(\frac{-\ln 2}{4.5 \times 10^9}\right) \doteq -1.540 \times 10^{-8}$ mg/a, after 10 000 a.

8. ^{24}Na half-life = 15h, $f(0) = y_0 = 2g$.
   (a) $\frac{1}{2} = e^{15k} \Rightarrow k = -\frac{\ln 2}{15} \Rightarrow f(t) = (2)e^{-\frac{\ln 2}{15}t} \Rightarrow f(t) = (2)2^{-\frac{t}{15}}$ after t hours.
   (b) $f(5) = (2)2^{-\frac{5}{15}} = 2^{\frac{2}{3}} \doteq 1.587$ g remaining after 5 h.
   (c) $f'(t) = (2)e^{-\frac{\ln 2}{15}t}\left(-\frac{\ln 2}{15}\right) \Rightarrow f'(5) = (2)e^{-\frac{5\ln 2}{15}}\left(-\frac{\ln 2}{15}\right) \doteq -0.073$ g/h after 5 h.
   (d) $0.4 = (2)2^{-\frac{t}{15}} \Rightarrow t = (-15)\frac{\ln 0.2}{\ln 2} = \frac{15\ln 5}{\ln 2} \doteq 34.8$ h, or about 34 h, 49 min for the sample to decompose to a mass of 0.4 g.

9. U-234 half-life is 2.5×10^5a

(a) $f(0) = y_0 = 10$; $\frac{1}{2} = e^{(2.5 \times 10^5)k} \Rightarrow k = \frac{-\ln 2}{2.5 \times 10^5}$

$$\Rightarrow f(t) = (10)e^{\left(\frac{-\ln 2}{2.5 \times 10^5}\right)t} \Rightarrow f(t) = (10)\ 2^{\left(\frac{-t}{2.5 \times 10^5}\right)}$$

$$f(10\ 000) = (10)\ 2^{\left(\frac{-1000}{2.5 \times 10^5}\right)} \doteq 9.972 \text{ mg after } 1000 \text{ a.}$$

(b) $7 = (10)\ 2^{\left(\frac{-t}{2.5 \times 10^5}\right)} \Rightarrow t = \frac{-(2.5 \times 10^5)\ln(0.7)}{\ln 2} \doteq 128\ 643$ a for the sample to decay to a mass of 7 g.

10. Bi-210 decayed to 33% of original mass in 8 d.

(a) $0.33 = e^{8k} \Rightarrow k = \frac{\ln 0.33}{8}$ but $\frac{1}{2} = e^{kt} \Rightarrow \frac{1}{2} = e^{\left(\frac{\ln 0.33}{8}\right)t} \Rightarrow$

$-\ln 2 = \frac{\ln 0.33}{8}t \Rightarrow t = \frac{8(-\ln 2)}{\ln 0.33} \doteq 5.00$d. The half-life of bi-210 is $\doteq$ 5 d.

(b) $f(12) = (1.00)(0.33)^{\frac{12}{8}} \doteq 0.19 \Rightarrow$ approximately 19% of the original mass remains after 12 d.

11. $P = \$1000$, $r = 0.16$, $t = 2$

(a) annually ($n = 1$): $A(2) = 1000(1 + 0.16)^2 \doteq \$1345.60$

(b) quarterly ($n = 4$): $A(2) = 1000(1 + 0.04)^8 \doteq \$1368.57$

(c) monthly ($n = 12$): $A(2) = 1000(1 + \frac{0.04}{3})^{12} \doteq \$1374.22$

(d) weekly ($n = 52$): $A(2) = 1000(1 + \frac{0.04}{13})^{104} \doteq \$1376.45$

(e) daily ($n = 365$): $A(2) = 1000(1 + \frac{0.16}{365})^{730} \doteq \$1377.03$

(f) continuously: $\lim_{n \to \infty} A(2) = \lim_{n \to \infty} 1000(1 + \frac{0.16}{n})^{2n}$ can be rewritten:

$$\lim_{n \to \infty} A(2) = 1000\left[\lim_{n \to 0}(1 + n)^{\frac{1}{n}}\right]^{0.32} = 1000e^{0.32} \doteq \$1377.13$$

12. $P = \$10\ 000$, $r = 0.10$, $t = 4$

(a) annually ($n = 1$): $A(4) = 10\ 000(1 + 0.10)^4 \doteq \$14\ 641.00$

(b) semianually ($n = 2$): $A(4) = 10\ 000(1 + 0.05)^8 \doteq \$14\ 774.55$

(c) monthly ($n = 12$): $A(4) = 10\ 000(1 + \frac{0.10}{12})^{48} \doteq \$14\ 893.54$

(d) daily ($n = 365$): $A(4) = 10\ 000(1 + \frac{0.10}{365})^{1\ 460} \doteq \$14\ 917.43$

(e) hourly ($n = 8\ 760$): $A(4) = 10\ 000(1 + \frac{0.10}{8\ 760})^{35\ 040} \doteq \$14\ 918.21$

(f) continuously: $A(4) = \lim_{n \to \infty} 10\ 000(1 + \frac{0.10}{n})^{4n}$ can be rewritten:

$$A(4) = 10\ 000 \left[\lim_{n \to 0} (1 + n)^{\frac{1}{n}}\right]^{0.40} = 10\ 000 e^{0.40} \doteq \$14\ 918.25$$

13. $A(t) = P_0(1.046\ 25)^{2t} = P_0(1.094\ 639\ 063)^t$ or
$A(t) = P_0(e)^{0.09t} = P_0(1.094\ 174\ 28)^t$, therefore, $9\frac{1}{4}$% compounded semiannually is the better investment.

14. $A(t) = P_0(e)^{0.085t}$ doubles when $2 = e^{0.085t} \Rightarrow t = \frac{\ln 2}{0.085} \doteq 8.155$ a, or about 8 a, 56 d to double.

15. $-\frac{d[N_2O_5]}{dt} = 0.0005[N_2O_5]$. Let $[N_2O_5] = y$, let $-0.0005 = k \Rightarrow \frac{dy}{dx} = ky \Rightarrow$
$y = y_0 e^{kt} \Rightarrow y = Ce^{kt}$

(a) $[N_2O_5](t) = Ce^{-0.0005t}$ after t seconds.

(b) $\frac{1}{2}C = Ce^{-0.0005t} \Rightarrow -0.0005t = \ln(\frac{1}{2}) \Rightarrow t = \frac{-\ln 2}{-0.0005} = 2000 \ln 2$
$\doteq 1386$ s for the initial concentration to be reduced by half.

16. ^{14}C half-life is 5570 years: $\frac{1}{2} = e^{5570k} \Rightarrow -\ln 2 = 5570k \Rightarrow k = \frac{\ln 2}{-5570}$
Thus $0.77 = e^{\frac{\ln 2}{-5570}t} \Rightarrow \ln 0.77 = \frac{(\ln 2)t}{-5570} \Rightarrow t = \frac{(-5570)(\ln 0.77)}{\ln 2} \doteq 2100$ a.
The parchment is approximately 2100 years old.

EXERCISE 8.6

1. (a) $y=(x^2+1)^2(x^2+x+1)^3>0$ for all $x\in R$ $\Rightarrow$

$\ln y=2\ln(x^2+1)+3\ln(x^2+x+1)\Rightarrow\frac{y'}{y}=\frac{2(2x)}{x^2+1}+\frac{3(2x+1)}{x^2+x+1}\Rightarrow$

$y'=(x^2+1)^2(x^2+x+1)^3\left[\frac{4x}{x^2+1}+\frac{6x+3}{x^2+x+1}\right]$

(b) $y=(x-1)^4(2x+3)^5(x^2-2x+3)^3\Rightarrow|y|=|x-1|^4|2x+3|^5|x^2-2x+3|^3\Rightarrow$

$\ln|y|=4\ln|x-1|+5\ln|2x+3|+3\ln|x^2-2x+3|\Rightarrow$

$\frac{y'}{y}=\frac{4}{x-1}+\frac{10}{2x+3}+\frac{6(x-1)}{x^2-2x+3}$

$\Rightarrow y'=(x-1)^4(2x+3)^5(x^2-2x+3)^3\left[\frac{4}{x-1}+\frac{10}{2x+3}+\frac{6x-6}{x^2-2x+3}\right]$

(c) $|y|=|e^{x^2}x^3(x^2+8)^4|\Rightarrow|y|=e^{x^2}|x|^3(x^2+8)^4\Rightarrow$

$\ln|y|=x^2+3\ln|x|+4\ln(x^2+8)\Rightarrow\frac{y'}{y}=2x+\frac{3}{x}+\frac{8x}{x^2+8}\Rightarrow$

$y'=e^{x^2}x^3(x^2+8)^4\left[2x+\frac{3}{x}+\frac{8x}{x^2+8}\right]$

(d) $y=\frac{(x+1)^3}{(x+2)^5(x+3)^7}\Rightarrow|y|=\frac{|x+1|^3}{|x+2|^5|x+3|^7}\Rightarrow$

$\ln|y|=3\ln|x+1|-5\ln|x+2|-7\ln|x+3|\Rightarrow\frac{y'}{y}=\frac{3}{x+1}-\frac{5}{x+2}-\frac{7}{x+3}\Rightarrow$

$y'=\frac{(x+1)^3}{(x+2)^5(x+3)^7}\left[\frac{3}{x+1}-\frac{5}{x+2}-\frac{7}{x+3}\right]$

(e) $y=\frac{x\sqrt{x+1}}{(x+2)(x^3+1)}$ (Domain: $(-1,\infty)$) $\Rightarrow|y|=\frac{|x|\sqrt{x+1}}{(x+2)(x^3+1)}\Rightarrow$

$\ln|y|=\ln|x|+\frac{1}{2}\ln(x+1)-\ln(x+2)-\ln(x^3+1)\Rightarrow$

$\frac{y'}{y}=\frac{1}{x}+\frac{1}{2(x+1)}-\frac{1}{x+2}-\frac{3x^2}{x^3+1}\Rightarrow$

$y'=\frac{x\sqrt{x+1}}{(x+2)(x^3+1)}\left[\frac{1}{x}+\frac{1}{2(x+1)}-\frac{1}{x+2}-\frac{3x^2}{x^3+1}\right]$

(f) $y=\sqrt{\frac{x^2+1}{x^2+4}}>0$ for all $x\in R$ $\Rightarrow\ln y=\frac{1}{2}\ln(x^2+1)-\frac{1}{2}\ln(x^2+4)\Rightarrow$

$\frac{y'}{y}=\frac{x}{x^2+1}-\frac{x}{x^2+4}\Rightarrow y'=\sqrt{\frac{x^2+1}{x^2+4}}\left[\frac{x}{x^2+1}-\frac{x}{x^2+4}\right]$

2. (a) $y=x^{x^2}\Rightarrow\ln y=\ln x^{x^2}\Rightarrow\ln y=x^2\ln x\Rightarrow\frac{y'}{y}=2x\ln x+x\Rightarrow y'=x^{x^2}(2x\ln x+x)$

(b) $y=x^{\sqrt{x}} \Rightarrow \ln y=\ln x^{\sqrt{x}} \Rightarrow \ln y=\sqrt{x}\ln x \Rightarrow$
$\frac{y'}{y}=\frac{\ln x}{2\sqrt{x}}+\frac{1}{\sqrt{x}}=\frac{1}{\sqrt{x}}(\frac{1}{2}\ln x+1) \Rightarrow y'=\frac{x^{\sqrt{x}}}{\sqrt{x}}(\frac{1}{2}\ln x+1)=x^{\sqrt{x}-\frac{1}{2}}(\frac{1}{2}\ln x+1)$

(c) $y=x^{\cos x} \Rightarrow \ln y=\ln x^{\cos x} \Rightarrow \ln y=(\cos x)\ln x \Rightarrow$
$\frac{y'}{y}=-(\sin x)\ln x+\frac{\cos x}{x} \Rightarrow y'=x^{\cos x}\left[-(\sin x)\ln x+\frac{\cos x}{x}\right]$

(d) $y=(\cos x)^x \Rightarrow \ln y=\ln(\cos x)^x \Rightarrow \ln y=x\ln(\cos x) \Rightarrow$
$\frac{y'}{y}=\ln(\cos x)+\frac{-x\sin x}{\cos x} \Rightarrow y'=(\cos x)^x[\ln(\cos x)-x\tan x]$

(e) $y=(\ln x)^x \Rightarrow \ln y=\ln(\ln x)^x \Rightarrow \ln y=x\ln(\ln x) \Rightarrow \frac{y'}{y}=\ln(\ln x)+\frac{x}{x\ln x} \Rightarrow$
$y'=(\ln x)^x\left[\ln(\ln x)+\frac{1}{\ln x}\right]$

(f) $y=(\cos x)^{\sin x} \Rightarrow \ln y=\ln(\cos x)^{\sin x} \Rightarrow \ln y=(\sin x)\ln(\cos x) \Rightarrow$
$\frac{y'}{y}=(\cos x)\ln(\cos x)+\frac{-\sin^2 x}{\cos x} \Rightarrow y'=(\cos x)^{\sin x}[(\cos x)\ln(\cos x)-(\sin x)\tan x]$

3. $y=x^x$, $(2,4) \Rightarrow \ln y=\ln x^x \Rightarrow \ln y=x\ln x \Rightarrow \frac{y'}{y}=\ln x+1 \Rightarrow y'=x^x(\ln x+1) \Rightarrow$
$y'(2)=2^2(\ln 2+1)=4\ln 2+4 \Rightarrow$ the tangent is $y-4=(4\ln 2+4)(x-2) \Rightarrow$
$(4\ln 2+4)x-y-4(2\ln 2+1)=0$

4. (a) $f(x)=x^{-\ln x} \Rightarrow \ln f(x)=\ln x^{-\ln x} \Rightarrow f(x)=(e^{\ln x})^{-\ln x}=e^{-(\ln x)^2} \Rightarrow$

$\lim_{x\to 0^+} x^{-\ln x}=\lim_{x\to 0^+} e^{-(\ln x)^2}=\lim_{x\to -\infty} e^{-x^2}$ (since $\ln x\to-\infty$ as $x\to 0^+$) $=0$

since $-x^2\to-\infty$ as $x\to\infty$; $\lim_{x\to\infty} x^{-\ln x}=\lim_{x\to\infty} e^{-(\ln x)^2}=\lim_{x\to\infty} e^{-x^2}$

(since $\ln x\to\infty$ as $x\to\infty$) $=0$ since $-x^2\to-\infty$ as $x\to\infty$.

(b) $y=x^{-\ln x} \Rightarrow \ln y=\ln x^{-\ln x} \Rightarrow \ln y=-(\ln x)\ln x=-(\ln x)^2 \Rightarrow$
$\frac{y'}{y}=\frac{-2\ln x}{x} \Rightarrow y'=x^{-\ln x}(x^{-1})(-2\ln x)=-2x^{-1-\ln x}(\ln x)=\frac{-2x^{-\ln x}(\ln x)}{x}$

(c) $f(x)$ increases on $(0,1)$ and $f(x)$ decreases on $(1,\infty)$.

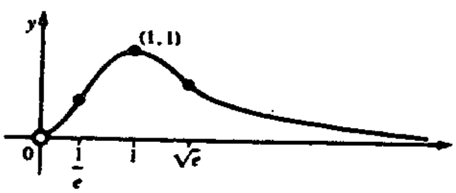
(d) $f(1)=1$ is the absolute maximum, by the first derivative test done in (c).

(e) $y''=\frac{2x^{-\ln x}}{x^2}\left[2(\ln x)^2+\ln x-1\right]=\frac{2x^{-\ln x}}{x^2}(2\ln x-1)(\ln x+1)$, $y''>0$ if

$0<x<\frac{1}{e}$ or if $x>\sqrt{e}$ and $y''<0$ if $\frac{1}{e}<x<\sqrt{e}$. Therefore, the graph is CU on

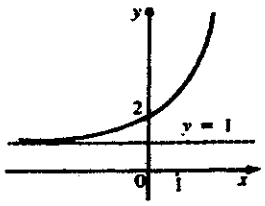
$(0,\frac{1}{e})$ and $(\sqrt{e},\infty)$ and CD on $(\frac{1}{e},\sqrt{e})$. The inflection points are $(\frac{1}{e},\frac{1}{e})$ and $(\sqrt{e},\sqrt[4]{e})$.

(f) $f(x)=x^{-\ln x}$

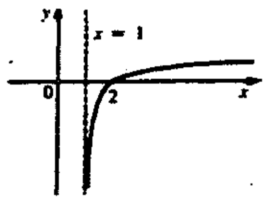


8.7 REVIEW EXERCISE

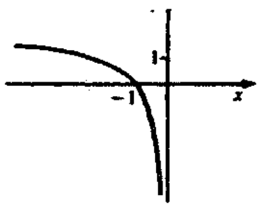
1. (a) $y = 1 + 2^x$



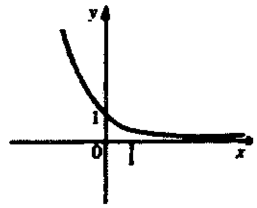
(b) $y = \log_{10}(x - 1)$



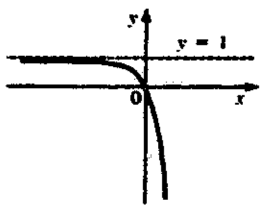
(c) $y = \ln(-x)$



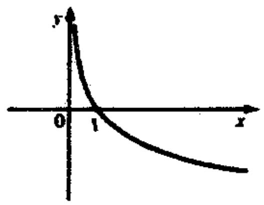
(d) $y = e^{-x}$



(e) $y = 1 - 10^x$



(f) $y = -\log_2 x$



2. (a) $\lim\limits_{x \to -\infty}(1 + 2^x) = 1 + \lim\limits_{x \to -\infty} 2^x = 1 + 0 = 1$

(b) $\lim\limits_{x \to 1^+} \log_{10}(x - 1) = -\infty$

(c) $\lim\limits_{x \to \infty} \ln(x^2 + x + 1) = \infty$

(d) $\lim\limits_{x \to -1^-} e^{\frac{2}{x+1}} = 0$, since $\frac{2}{x+1} \to -\infty$ as $x \to -1^-$

(e) $\lim\limits_{x \to \frac{\pi}{2}^-} e^{\tan x} = \infty$, since $\tan x \to \infty$ as $x \to \frac{\pi}{2}^-$

(f) $\lim\limits_{x \to 10^-} \ln(10 - x) = -\infty$

3. (a) $y = 1 + \ln(x + 2)$; Domain: $(-2, \infty)$; Range: R; Asymptote: $x = -2$

(b) $y = 1 + 3e^{2x}$; Domain: R; Range: $(1, \infty)$; Asymptote: $y = 1$

(c) $y = 10 - e^{-x}$; Domain: R; Range: $(-\infty, 10)$; Asymptote: $y = 10$

(d) $y = \ln(1 - 2x)$; Domain: $(-\infty, \frac{1}{2})$; Range: R; Asymptote: $x = \frac{1}{2}$

4. (a) $\ln 1 = 0$

(b) $e^{\ln 10} = 10$

(c) $e^{3\ln 2} = 2^3 = 8$

(d) $\ln\left(\frac{1}{e}\right) = -1$

8.7 Review Exercise

5. (a) $\ln x = \frac{1}{2} \Rightarrow x = e^{\frac{1}{2}} = \sqrt{e} \doteq 1.648\ 721$

   (b) $e^{x} = 7 \Rightarrow x = \ln 7 \doteq 1.945\ 910$

   (c) $e^{5-3x} = 2 \Rightarrow 5 - 3x = \ln 2 \Rightarrow x = \frac{1}{3}(5 - \ln 2) \doteq 1.435\ 618$

   (d) $\ln(4x + 7) = 4 \Rightarrow 4x + 7 = e^{4} \Rightarrow x = \frac{1}{4}(e^{4} - 7) \doteq 11.899\ 538$

6. (a) $2\ln x + 3\ln(1 + x) - 4\ln(2 + x) = \ln x^{2} + \ln(1 + x)^{3} - \ln(2 + x)^{4} = \ln\left[\frac{x^{2}(1 + x)^{3}}{(2 + x)^{4}}\right]$

   (b) $\frac{1}{2}\ln x - 2\ln(x^{2} + x + 1) = \ln\sqrt{x} - \ln(x^{2} + x + 1)^{2} = \ln\left[\frac{\sqrt{x}}{(x^{2} + x + 1)^{2}}\right]$

7. (a) $f(x) = \ln(x^{2} + 1) \Rightarrow f'(x) = \frac{2x}{x^{2} + 1}$

   (b) $f(x) = e^{x^{3}} \Rightarrow f'(x) = 3x^{2}e^{x^{3}}$

   (c) $f(x) = \sqrt{x}\,e^{x} \Rightarrow f'(x) = \frac{e^{x}}{2\sqrt{x}} + \sqrt{x}\,e^{x} = \frac{e^{x}}{2\sqrt{x}}(1 + 2x)$

   (d) $f(x) = \frac{\ln x}{x^{2}} \Rightarrow f'(x) = \frac{x^{2}(\frac{1}{x}) - 2x\ln x}{x^{4}} = \frac{1 - 2\ln x}{x^{3}}$

   (e) $y = x^{4} - 4^{x} \Rightarrow y' = 4x^{3} - 4^{x}\ln 4$

   (f) $y = \ln\sqrt{\frac{2x + 3}{4x - 5}} = \frac{1}{2}\ln(2x + 3) - \frac{1}{2}\ln(4x - 5) \Rightarrow$

   $y' = \frac{1}{2x + 3} - \frac{2}{4x - 5} = \frac{-11}{(2x + 3)(4x - 5)}$

   (g) $y = \sin(e^{2x}) \Rightarrow y' = 2e^{2x}\cos(e^{2x})$

   (h) $y = e^{2\sin x} \Rightarrow y' = (2\cos x)e^{2\sin x}$

   (i) $y = \log_{10}(1 - x + x^{3}) \Rightarrow y' = \frac{3x^{2} - 1}{(\ln 10)(1 - x + x^{3})}$

   (j) $y = e^{x}\ln x \Rightarrow y' = e^{x}\ln x + \frac{e^{x}}{x} = e^{x}(\ln x + \frac{1}{x})$

   (k) $y = \frac{e^{x^{2}}}{x^{2}} \Rightarrow y' = \frac{2x^{3}e^{x^{2}} - 2xe^{x^{2}}}{x^{4}} = \frac{2e^{x^{2}}}{x^{3}}(x^{2} - 1)$

   (l) $y = \sqrt{1 + (\ln x)^{4}} \Rightarrow y' = \frac{4(\ln x)^{3}}{2\sqrt{1 + (\ln x)^{4}}}\left(\frac{1}{x}\right) = \frac{2(\ln x)^{3}}{x\sqrt{1 + (\ln x)^{4}}}$

8. (a) $y = 2^{x}$, $(0,1) \Rightarrow y' = 2^{x}(\ln 2) \Rightarrow y'(0) = \ln 2 \Rightarrow$ the tangent is
   $y - 1 = (\ln 2)(x - 0)$ which is $(\ln 2)x - y + 1 = 0$

   (b) $y = \frac{\ln x}{x}$, $(1,0) \Rightarrow y' = \frac{\ln x}{-x^{2}} + \frac{1}{x^{2}} = \frac{1 - \ln x}{x^{2}} \Rightarrow y'(1) = 1 \Rightarrow$ the tangent is
   $y - 0 = 1(x - 1)$ which is $x - y - 1 = 0$

9. $f(x)=e^{-x}\cos 2x \Rightarrow f'(x)=-e^{-x}\cos 2x-2e^{-x}\sin 2x=e^{-x}(-\cos 2x-2\sin x)$

$f''(x)=e^{-x}\cos 2x+2e^{-x}\sin 2x+2e^{-x}\sin 2x-4e^{-x}\cos 2x=e^{-x}(4\sin 2x-3\cos 2x)$

$f''(0)=1\left[4(0)-3(1)\right]=-3$

10. (a) $|y|=|x|^5e^x\sqrt{x^2-x+1} \Rightarrow \ln|y|=\ln|x|^5+\ln e^x+\ln\sqrt{x^2-x+1}$

$\Rightarrow \ln|y|=5\ln|x|+x+\frac{1}{2}\ln(x^2-x+1) \Rightarrow \frac{y'}{y}=\frac{5}{x}+1+\frac{2x-1}{2(x^2-x+1)}$

$\Rightarrow y'=x^5e^x\sqrt{x^2-x+1}\left[\frac{5}{x}+1+\frac{2x-1}{2(x^2-x+1)}\right]$

(b) $y=\sqrt{x}^x \Rightarrow \ln y=\ln\sqrt{x}^x \Rightarrow \ln y=x\ln\sqrt{x}=\frac{1}{2}x\ln x \Rightarrow$

$\frac{y'}{y}=\frac{1}{2}+\frac{1}{2}\ln x=\frac{1}{2}(\ln x+1) \Rightarrow y'=\frac{\sqrt{x}^x}{2}(\ln x+1)$

11. $f(x)=2x^2-\ln x \Rightarrow f'(x)=4x-\frac{1}{x}>0$ when $4x^2>1 \Rightarrow x^2>\frac{1}{4} \Rightarrow$ f increases on $(\frac{1}{2},\infty)$; f decreases on $(0,\frac{1}{2})$.

12. $g(x)=e^x-x \Rightarrow g'(x)=e^x-1>0$ when $e^x>1 \Rightarrow$ g increases on $(0,\infty)$ and g decreases on $(-\infty,0) \Rightarrow g(0)=1$ is an absolute minimum.

13. (a) $\boxed{y=e^x+e^{-2x}}$ A. Domain: R

B. Intercepts: y-intercept is 2, no x-intercept

C. No Symmetry

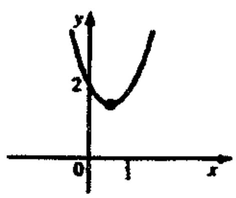
D. Asymptotes: No V.A., $\lim_{x\to\pm\infty} e^x+e^{-2x}=\infty \Rightarrow$ No H.A.

E. Inc/Dec: $y'=e^x-2e^{-2x} \Rightarrow$ y decreases on $(-\infty,\frac{\ln 2}{3})$, y increases on $(\frac{\ln 2}{3},\infty)$

F. $f(\frac{1}{3}\ln 2)=\sqrt[3]{2}+\sqrt[3]{\frac{1}{4}}=\frac{3}{2}\sqrt[3]{2}$ is an absolute minimum

G. Concavity: $y''=e^x+4e^{-2x}>0 \Rightarrow$ CU on $(-\infty,\infty)$; No Inflection point.

H.



(b) $\boxed{y=\ln(1+x^2)}$ A. Domain: R

B. Intercepts: (0,0)

C. Symmetry: $f(-x)=f(x) \Rightarrow$ y is even, symmetric about the y-axis

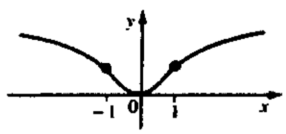
D. Asymptotes: No V.A., $\lim_{x\to\pm\infty}\ln(1+x^2)=\infty \Rightarrow$ No H.A.

E. Inc/Dec: $y' = \frac{2x}{1+x^2} \Rightarrow$ y decreases on $(-\infty, 0)$, y increases on $(0, \infty)$

F. $f(0) = 0$ is an absolute minimum

G. Concavity: $y'' = \frac{2(1+x^2) - 2x(2x)}{(1+x^2)^2} = \frac{2(1-x^2)}{(1+x^2)^2} > 0$ when $x^2 < 1 \Rightarrow$ f is CU on $(-1, 1)$, CD on $(-\infty, -1)$ and $(1, \infty)$; $(\pm 1, \ln 2)$ are inflection points.

H.



14. (a) $\log_2 93.5 = \frac{\ln 93.5}{\ln 2} \doteq 6.546\ 894$

(b) $\ln b = \log_e b = \frac{\log_a b}{\log_a e}$ Let $a = b \Rightarrow \ln b = \frac{\log_b b}{\log_b e} = \frac{1}{\log_b e}$

$\frac{d}{dx}\log_b x = \frac{1}{x \ln b} = \frac{1}{x\left(\frac{1}{\log_b e}\right)} = \frac{1}{x}\log_b e$

15. $f(0) = y_0 = 800$, $f(4) = 7200$

(a) $f(t) = y_0 e^{kt} \Rightarrow$ at $t = 4$, $7200 = 800e^{4k} \Rightarrow k = \frac{1}{4}\ln\frac{7200}{800} = \frac{1}{4}\ln 9 = \frac{1}{2}\ln 3$

$f(t) = 800e^{(\frac{1}{2}\ln 3)t} = 800(3)^{\frac{t}{2}}$ after t hours.

(b) $f(6) = 800(3)^3 = 21\ 600$ bacteria after 6 h.

(c) $f'(t) = 800e^{(\frac{1}{2}\ln 3)t}(\frac{1}{2}\ln 3) \Rightarrow f'(6) = 800e^{(3\ln 3)}(\frac{1}{2}\ln 3) \doteq 11\ 865$ bacteria/hour after 6 h.

(d) $20\ 000 = 800e^{(\frac{1}{2}\ln 3)t} \Rightarrow t = \frac{2\ln 25}{\ln 3} \doteq 5.86$ h, or about 5 h 51 min for the population to reach 20 000 bacteria.

16. $f(0) = y_0 = 1$g, $f(10) = 1.2$g

(a) $f(t) = y_0 e^{kt} \Rightarrow$ at $t = 10$, $1.2 = e^{10k} \Rightarrow k = \frac{1}{10}\ln 1.2 \Rightarrow f(t) = e^{(\frac{1}{10}\ln 1.2)t}$

$\Rightarrow f(t) = (1.2)^{\frac{t}{10}}$ after t hours.

(b) $f(24) = (1.2)^{2.4} \doteq 1.549$g after 24 h.

(c) $2 = e^{\frac{t}{10}\ln 1.2} \Rightarrow t = \frac{10\ln 2}{\ln 1.2} \doteq 38$ h; the doubling time is about 38 h.

17. Pb-214 half-life is 26.8 min, $f(0) = y_0 = 15\text{g}$

(a) $f(t) = y_0 e^{kt} \Rightarrow \frac{1}{2} = e^{26.8k} k = \frac{-\ln 2}{26.8} \Rightarrow f(t) = 15e^{\frac{-\ln 2}{26.8}t} = 15(2)^{\frac{-t}{26.8}}$ after t minutes.

(b) $f(60) = 15(2)^{\frac{-60}{26.8}} \doteq 3.178$ g after 1 h.

(c) $f'(t) = 15e^{\frac{-\ln 2}{26.8}t}\left[\frac{-\ln 2}{26.8}\right] \Rightarrow f'(60) = 15e^{\frac{-\ln 2}{26.8}(60)}\left[\frac{-\ln 2}{26.8}\right] \doteq -0.082\ 193$ g/min after 1 h.

(d) $1 = 15e^{\frac{-\ln 2}{26.8}t} \Rightarrow t = \frac{26.8(\ln 15)}{\ln 2} \doteq 105$ min, or about 1 h 45 min for the sample to decompose to 1 g.

18. (a) $P = \$5000$, $r = 0.08$, $t = 3$

(i) annually $(n = 1)$: $A(3) = 5000(1 + 0.08)^3 \doteq \$6298.56$

(ii) semiannually $(n = 2)$: $A(3) = 5000(1 + 0.04)^6 \doteq \$6326.60$

(iii) daily $(n = 365)$: $A(3) = 5000(1 + \frac{0.08}{365})^{1095} \doteq \$6356.08$

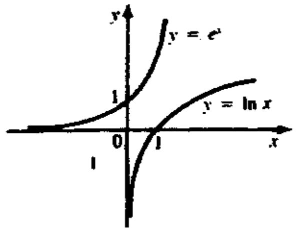
(iv) continuously: $\lim_{n \to \infty} A(3) = \lim_{n \to \infty} 5000(1 + \frac{0.08}{n})^{3n}$ can be rewritten:

$$\lim_{n \to \infty} A(3) = 5000\left[\lim_{n \to 0}(1 + n)^{\frac{1}{n}}\right]^{0.24} = 5000e^{0.24} \doteq \$6356.25$$

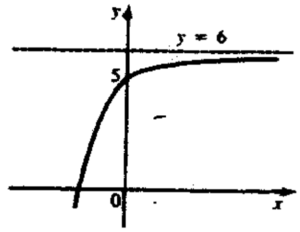
(b) $5000e^{0.8t} = 8000 \Rightarrow t = \frac{\ln 1.6}{0.08} \doteq 5.9$ a for the investment to reach \$8000 if the interest is compounded continuously.

8.8 CHAPTER 8 TEST

1. $f(x) = e^x$ and $g(x) = \ln x$



2. $y = 6 - e^{-x}$; Domain: Real; Range: $(-\infty, 6)$; Asymptote: $y = 6$



3. (a) $\lim\limits_{x \to 8^+} \ln(x-8) = -\infty$, since $x - 8 \to 0$ as $x \to 8^+$

   (b) $\lim\limits_{x \to \infty} e^{-x^2} = 0$, since $-x^2 \to -\infty$ as $x \to \infty$

4. $e^{-2\ln 3} = e^{\ln 3^{-2}} = 3^{-2} = \frac{1}{9}$

5. $e^{1-2x} = 5 \Rightarrow 1 - 2x = \ln 5 \Rightarrow x = \frac{1}{2}(1 - \ln 5)$

6. (a) $y = x^2 \ln(x^3 + 2x - 1) \Rightarrow y' = \dfrac{x^2(3x^2+2)}{x^3+2x-1} + 2x\ln(x^3+2x-1)$

   (b) $y = \dfrac{e^{4x}}{x^2+1} \Rightarrow y' = \dfrac{4e^{4x}(x^2+1) - 2xe^{4x}}{(x^2+1)^2} = \dfrac{2e^{4x}}{(x^2+1)^2}(2x^2 - x + 2)$

   (c) $y = e^{\tan\sqrt{x}} \Rightarrow y' = \dfrac{1}{2\sqrt{x}}(\sec^2\sqrt{x})e^{\tan\sqrt{x}}$

   (d) $y = 2^{-\frac{1}{x}} \Rightarrow \dfrac{\ln 2}{x^2}2^{-\frac{1}{x}}$

   (e) $y = x^{x^3} \Rightarrow \ln y = x^3 \ln x \Rightarrow \dfrac{y'}{y} = 3x^2\ln x + x^2 \Rightarrow y' = x^{x^3+2}(3\ln x + 1)$

7. $f(0) = y_0 = 1000$, $f(1) = 7000$

   (a) $f(t) = y_0 e^{kt} \Rightarrow$ at $t = 1$, $7000 = 1000e^k \Rightarrow k = \ln 7 \Rightarrow f(t) = 1000e^{(\ln 7)t}$ $\Rightarrow f(t) = 1000(7)^t$ after t hours.

   (b) $f(3) = 1000(7)^3 = 343\ 000$ bacteria after 3h.

   (c) $f'(t) = 1000(7)^t(\ln 7) \Rightarrow f'(3) = 1000(7)^3(\ln 7) \doteq 667\ 447$ bacteria/h after 3 h.

   (d) $10\ 000 = 1000(7)^t \Rightarrow 10 = 7^t \Rightarrow t = \dfrac{\ln 10}{\ln 7} \doteq 1.18$ h, or about 1 h, 11 min for the population to reach 10 000 bacteria.

8. $A = Pe^{rt} \Rightarrow 2 = e^{0.09t} \Rightarrow t = \frac{\ln 2}{0.09} \doteq 7.7$ a for the investment to double.

9. (a). $\boxed{y = \ln(9 - x^2)}$ (a) Domain: $(-3, 3)$

(b) Intercepts: y-intercept is $2\ln 3$, x-intercepts are $\pm 2\sqrt{2}$

(c) Symmetry: $f(-x) = f(x) \Rightarrow$ y is even, symmetric about the y-axis

(d) Asymptotes: $\lim_{x \to 3^-} \ln(9 - x^2) = \lim_{x \to -3^+} \ln(9 - x^2) = -\infty \Rightarrow x = \pm 3$ are V.A., No H.A.

(e) Inc/Dec: $y' = \frac{-2x}{9 - x^2} \Rightarrow$ y decreases on $(0, 3)$, y increases on $(-3, 0)$

(f) $f(0) = 2\ln 3$ is an absolute maximum

(g) Concavity: $y'' = \frac{-2(9 - x^2) + 2x(-2x)}{(9 - x^2)^2} = \frac{-2(x^2 + 9)}{(9 - x^2)^2} \Rightarrow$ CD on $(-3, 3)$;

No inflection points.

(h)