Review and Preview to Chapter 1

EXERCISE 1

1. (a) $x^2 - x - 2 = (x-2)(x+1)$ (b) $x^2 - 9x + 14 = (x-2)(x-7)$
(c) $x^2 + 7x + 12 = (x+3)(x+4)$ (d) $2x^2 - x - 1 = (2x+1)(x-1)$
(e) $5x^2 + 13x + 6 = (5x+3)(x+2)$ (f) $6y^2 - 11y + 3 = (3y-1)(2y-3)$
(g) $t^3 + 2t^2 - 3t = t(t-1)(t+3)$ (h) $3x^4 + 7x^3 + 2x^2 = x^2(3x+1)(x+2)$

2. (a) $4x^2 - 25 = (2x+5)(2x-5)$ (b) $x^3 - 1 = (x-1)(x^2+x+1)$
(c) $t^3 + 64 = (t+4)(t^2-4t+16)$ (d) $y^3 - 9y = y(y+3)(y-3)$
(e) $8c^3 - 27d^3 = (2c-3d)(4c^2+6cd+9d^2)$ (f) $x^6 + 8 = (x^2+2)(x^4-2x^2+4)$
(g) $x^4 - 16 = (x+2)(x-2)(x^2+4)$ (h) $r^8 - 1 = (r+1)(r-1)(r^2+1)(r^4+1)$

3. (a) $x^3 - x^2 - 16x + 16 = x^2(x-1) - 16(x-1) = (x^2-16)(x-1) = (x+4)(x-4)(x-1)$

(b) $x^3 - 7x + 6$; $x-1$ is a factor, so :

$$\begin{array}{r|l} & x^2 + x - 6 \\ \hline x-1 & x^3 \qquad - 7x + 6 \\ & \underline{x^3 - x^2} \\ & \quad x^2 - 7x \\ & \quad \underline{x^2 - x} \\ & \qquad -6x + 6 \\ & \qquad \underline{-6x + 6} \end{array}$$

Thus $x^3 - 7x + 6 = (x-1)(x^2+x-6)$
$= (x-1)(x+3)(x-2)$

(c) $x^3 + 5x^2 - 2x - 24$; $x-2$ is a factor, so :

$$\begin{array}{r|l} & x^2 + 7x + 12 \\ \hline x-2 & x^3 + 5x^2 - 2x - 24 \\ & \underline{x^3 - 2x^2} \\ & \quad 7x^2 - 2x \\ & \quad \underline{7x^2 - 14x} \\ & \qquad 12x - 24 \\ & \qquad \underline{12x - 24} \end{array}$$

$x^3 + 5x^2 - 2x - 24 = (x-2)(x^2+7x+12)$
$= (x-2)(x+3)(x+4)$

(d) $x^3 + 2x^2 - 11x - 12$; $x-3$ is a factor, so :

$$\begin{array}{r|l} & x^2 + 5x + 4 \\ \hline x-3 & x^3 + 2x^2 - 11x - 12 \\ & \underline{x^3 - 3x^2} \\ & \quad 5x^2 - 11x \\ & \quad \underline{5x^2 - 15x} \\ & \qquad 4x - 12 \\ & \qquad \underline{4x - 12} \end{array}$$

Thus $x^3 + 2x^2 - 11x - 12 = (x-3)(x^2+5x+4)$
$= (x-3)(x+1)(x+4)$

(e) $4x^3+12x^2+5x-6$; $x+2$ is a factor, so:

$$\begin{array}{r|l} & 4x^2+4x-3 \\ \hline x+2 & 4x^3+12x^2+5x-6 \\ & \underline{4x^3+8x^2} \\ & \quad 4x^2+5x \\ & \quad \underline{4x^2+8x} \\ & \qquad -3x-6 \\ & \qquad \underline{-3x-6} \end{array}$$

Thus $4x^3+12x^2+5x-6=(x+2)(4x^2+4x-3)$
$=(x+2)(2x-1)(2x+3)$

(f) $x^4-3x^3-7x^2+27x-18=x^4-7x^2-18-3x^3+27x=(x^2+2)(x^2-9)-3x(x^2-9)$
$=(x^2-9)(x^2+2-3x)=(x+3)(x-3)(x-2)(x-1)$

4. (a) $x^{\frac{5}{2}}-x^{\frac{1}{2}}=x^{\frac{1}{2}}(x^2-1)=x^{\frac{1}{2}}(x-1)(x+1)$

(b) $x+5+6x^{-1}=x^{-1}(x^2+5x+6)=x^{-1}(x+2)(x+3)$

(c) $x^{\frac{3}{2}}+2x^{\frac{1}{2}}-8x^{-\frac{1}{2}}=x^{-\frac{1}{2}}(x^2+2x-8)=x^{-\frac{1}{2}}(x+4)(x-2)$

(d) $2x^{\frac{7}{2}}-2x^{\frac{1}{2}}=2x^{\frac{1}{2}}(x^3-1)=2x^{\frac{1}{2}}(x-1)(x^2+x+1)$

(e) $1+2x^{-1}+x^{-2}=x^{-2}(x^2+2x+1)=x^{-2}(x+1)^2$

(f) $(x^2+1)^{\frac{1}{2}}+3(x^2+1)^{-\frac{1}{2}}=(x^2+1)^{-\frac{1}{2}}(x^2+1+3)=(x^2+1)^{-\frac{1}{2}}(x^2+4)$

EXERCISE 2

1. (a) $\dfrac{\sqrt{x}-3}{x-9}\times\dfrac{\sqrt{x}+3}{\sqrt{x}+3}=\dfrac{x-9}{(x-9)(\sqrt{x}+3)}=\dfrac{1}{\sqrt{x}+3}$

(b) $\dfrac{\frac{1}{\sqrt{x}}-1}{x-1}\times\dfrac{\frac{1}{\sqrt{x}}+1}{\frac{1}{\sqrt{x}}+1}=\dfrac{\frac{1}{x}-1}{\sqrt{x}+x-\frac{1}{\sqrt{x}}-1}$

$=\dfrac{\frac{1}{x}-1}{-\sqrt{x}(\frac{1}{x}-1)-x(\frac{1}{x}-1)}=\dfrac{-1}{x+\sqrt{x}}$

(c) $\dfrac{x\sqrt{x}-8}{x-4}\times\dfrac{x\sqrt{x}+8}{x\sqrt{x}+8}=\dfrac{x^3-64}{(x-4)(x\sqrt{x}+8)}=\dfrac{(x-4)(x^2+4x+16)}{(x-4)(x\sqrt{x}+8)}=\dfrac{x^2+4x+16}{x\sqrt{x}+8}$

(d) $\dfrac{\sqrt{2+h}+\sqrt{2-h}}{h} \times \dfrac{\sqrt{2+h}-\sqrt{2-h}}{\sqrt{2+h}-\sqrt{2-h}} = \dfrac{2+h-2+h}{h\left(\sqrt{2+h}-\sqrt{2-h}\right)} = \dfrac{2h}{h\left(\sqrt{2+h}-\sqrt{2-h}\right)}$

$= \dfrac{2}{\sqrt{2+h}-\sqrt{2-h}}$

(e) $\left(\sqrt{x^2+3x+4}-x\right)\left(\dfrac{\sqrt{x^2+3x+4}+x}{\sqrt{x^2+3x+4}+x}\right) = \dfrac{x^2+3x+4-x^2}{\sqrt{x^2+3x+4}+x} = \dfrac{3x+4}{\sqrt{x^2+3x+4}+x}$

(f) $\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)\left(\dfrac{\sqrt{x^2+x}+\sqrt{x^2-x}}{\sqrt{x^2+x}+\sqrt{x^2-x}}\right) = \dfrac{x^2+x-x^2+x}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

$= \dfrac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

2. (a) $\dfrac{1}{\sqrt{x+1}-1} \times \dfrac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \dfrac{\sqrt{x+1}+1}{x+1-1} = \dfrac{\sqrt{x+1}+1}{x}$

(b) $\dfrac{4}{\sqrt{x+2}+\sqrt{x}} \times \dfrac{\sqrt{x+2}-\sqrt{x}}{\sqrt{x+2}-\sqrt{x}} = \dfrac{4\sqrt{x+2}-4\sqrt{x}}{x+2-x} = 2\left(\sqrt{x+2}-\sqrt{x}\right)$

(c) $\dfrac{x}{\sqrt{x^2+1}+x} \times \dfrac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}-x} = \dfrac{x\sqrt{x^2+1}-x^2}{x^2+1-x^2} = x\left(\sqrt{x^2+1}-x\right)$

(d) $\dfrac{x^2}{\sqrt{x+1}-\sqrt{x-1}} \times \dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} = \dfrac{x^2\sqrt{x+1}+x^2\sqrt{x-1}}{x+1-x+1}$

$= \frac{1}{2}x^2\left(\sqrt{x+1}+\sqrt{x-1}\right)$

EXERCISE 1.1

1. (a) $y = 4x$, slope $= 4$

(b) $y = 3x - 5$, slope $= 3$

(c) $f(x) = \frac{1}{3}x - 2$, slope $= \frac{1}{3}$

(d) $f(x) = 2 - 3x$, slope $= -3$

(e) $f(x) = \frac{1}{2}(1 - x) = \frac{1}{2} - \frac{1}{2}x$, slope $= -\frac{1}{2}$

(f) $x + 2y = 3 \Rightarrow y = -\frac{1}{2}x + \frac{3}{2}$, slope $= -\frac{1}{2}$

2. $(-3,5), (4,-5)$, $m = \dfrac{-5 - (5)}{4 - (-3)} = -\frac{10}{7}$, $y - 5 = -\frac{10}{7}(x - (-3)) \Rightarrow y = -\frac{10}{7}x + \frac{6}{7}$
$\Rightarrow 10x + 7y - 5 = 0$

3. $(-4,-2), (2,10)$, $m = \dfrac{10 - (-2)}{2 - (-4)} = 2$, $y - 10 = 2(x - 2) \Rightarrow f(x) = 2x + 6$

4. $y = 16 + 3x \Rightarrow$ slope $= 3$

(a) if x increases by 4, y increases by $3 \times 4 = 12$

(b) if x decreases by 2, y increases by $3(-2) = -6$ or decreases by 6

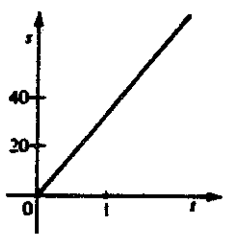
5. $y = \dfrac{1 - x}{2} \Rightarrow$ slope $= -\frac{1}{2}$

(a) if x increases by 6, y increases by $-\frac{1}{2}(6) = -3$ or decreases by 3

(b) if x decreases by 4, y increases by $-\frac{1}{2}(-4) = 2$

6. $s = 140$ km, $t = 4$ h ; slope $= \frac{140}{4} = 35 \Rightarrow s = 35t$.

The slope of the line represents the speed of the car.



7. $P(1,3)$ lies on $y = 2x + x^2$

(a) $Q(x, x^2 + 2x)$

(i) $x = 2$, slope $= \dfrac{(2)^2 + 2(2) - 3}{2 - 1} = 5$

(ii) $x = 1.5$, slope $= 4.5$

(iii) $x = 1.1$, slope $= 4.1$

(iv) $x = 1.01$, slope $= 4.01$

(v) $x = 1.001$, slope $= 4.001$

(vi) $x = 0$, slope $= 3$

(vii) $x = 0.5$, slope $= 3.5$

(viii) $x = 0.9$, slope $= 3.9$

(ix) $x = 0.99$, slope $= 3.99$

(x) $x = 0.999$, slope $= 3.999$

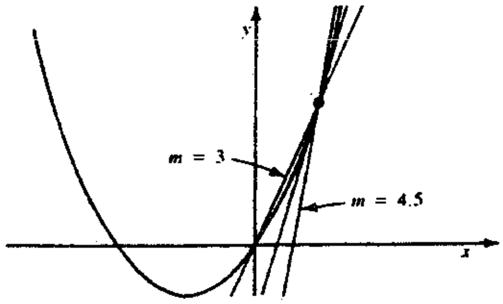
(b) Slope of the tangent line at $P(1,3)$ is 4.

(c) $y - 3 = 4(x - 1)$

$y = 4x - 1$ or $4x - y - 1 = 0$

Exercise 1.1

(d)



8. P(2,0) lies on $y = -x^2 + 6x - 8$

(a) $Q(x, -x^2 + 6x - 8)$ (i) $x = 3$, slope $= \dfrac{-(3)^2 + 6(3) - 8 - 0}{3 - 2} = 1$

(ii) $x = 2.5$, slope $= 1.5$ (iii) $x = 2.1$, slope $= 1.9$ (iv) $x = 2.01$, slope $= 1.99$

(v) $x = 1$, slope $= 3$ (vi) $x = 1.5$, slope $= 2.5$ (vii) $x = 1.9$, slope $= 2.1$

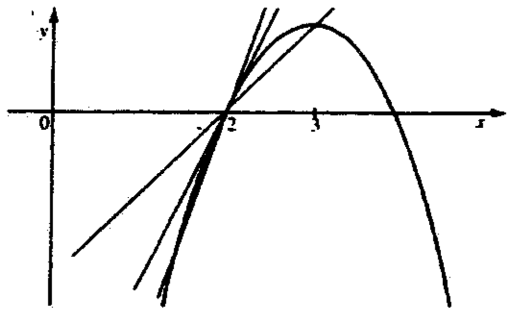
(viii) $x = 1.99$, slope $= 2.01$

(b) Slope of the tangent line at P(2,0) is 2.

(c) $y - 0 = 2(x - 2)$

$y = 2x - 4$ or $2x - y - 4 = 0$

(d)



9. $P(1,\frac{1}{4})$ lies on $y = \frac{1}{4}x^3$

(a) $Q(x,\frac{1}{4}x^3)$ (i) $x = 2$, slope $= \dfrac{\frac{1}{4}(2)^3 - \frac{1}{4}}{2 - 1} = 1.75$

(ii) $x = 1.5$, slope $= 1.1875$ (iii) $x = 1.1$, slope $= 0.8275$

(iv) $x = 1.01$, slope $= 0.757525$ (v) $x = 1.001$, slope $= 0.750075$

(vi) $x = 0$, slope $= 0.25$ (vii) $x = 0.5$, slope $= 0.4375$

(viii) $x = 0.9$, slope $= 0.6775$ (ix) $x = 0.99$, slope $= 0.742525$

(x) $x = 0.999$, slope $= 0.749925$

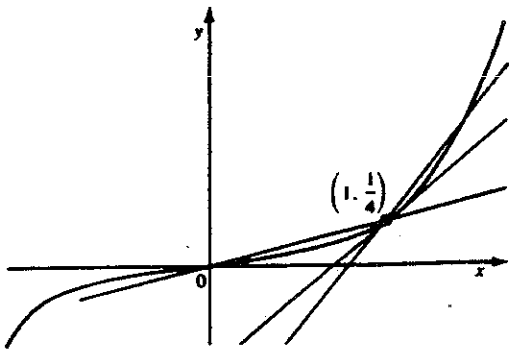
(b) Slope of the tangent line at $P(1,\frac{1}{4})$ is $0.75 = \frac{3}{4}$

(c) $y - \frac{1}{4} = \frac{3}{4}(x - 1)$

$y = \frac{3}{4}x - \frac{1}{2}$ or $3x - 4y - 2 = 0$

Exercise 1.1

(d)



10. P(0.5,2) lies on $y = \frac{1}{x}$

(a) $Q(x, \frac{1}{x})$

(i) $x = 2$, slope $= \dfrac{\frac{1}{2} - 2}{2 - \frac{1}{2}} = -1$

(ii) $x = 1$, slope $= -2$

(iii) $x = 0.9$, slope $= -2.2222$

(iv) $x = 0.8$, slope $= -2.5$

(v) $x = 0.7$, slope $= -2.8571$

(vi) $x = 0.6$, slope $= -3.3333$

(vii) $x = 0.55$, slope $= -3.6364$

(viii) $x = 0.51$, slope $= -3.921$ b6

(ix) $x = 0.45$, slope $= -4.4444$

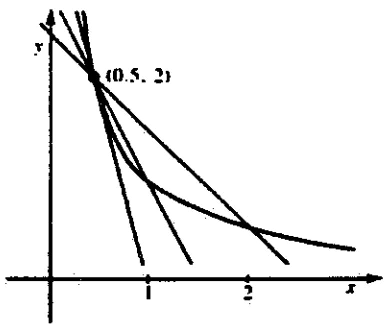
(x) $x = 0.49$, slope $= -4.0816$

(b) Slope of the tangent line at P(0.5,2) is -4.

(c) $y - 2 = -4(x - 0.5)$

$y = -4x + 4$ or $4x + y - 4 = 0$

(d)

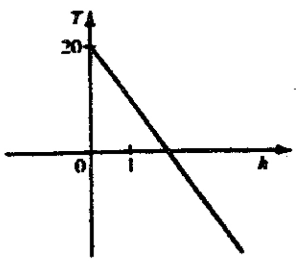


11. $T_0 = 20°$, h is in metres.

$m = \dfrac{\Delta T}{\Delta h} = -\dfrac{1}{100}$ °/m

$T = 20 - \dfrac{1}{100} h$ (h in metres) or $T = 20 - 10h$ (h in kilometres)

The slope represents the rate of increase of the temperature with increasing altitude.



Exercise 1.1

12. (500, 800), (650, 1400)

(a) slope $= \dfrac{650-500}{1400-800} = \dfrac{1}{4}$, $C - 500 = \frac{1}{4}(d-800) \rightarrow C = \frac{1}{4}d + 300$

(b) $C(2000) = \frac{1}{4}(2000) + 300 = \$800$

(c) The slope represents the cost per kilometre of driving a car.

(d) $d = 0$, $C = \$300$. This is reasonable (insurance, license, depreciation, ...)

(e) A linear function is suitable since total cost is fixed expenses plus per kilometre expenses.

Exercise 1.2

1. From the graph (a) $\lim_{x \to 3} f(x) = 1$ (b) $\lim_{x \to 2} f(x) = 0$

(c) $\lim_{x \to -1} f(x) = 1$ (d) $\lim_{x \to 4} f(x)$ does not exist.

2. (a) $\lim_{x \to 2} x^3 = 2^3 = 8$ (b) $\lim_{x \to \pi} x = \pi$ (c) $\lim_{x \to 8} 3 = 3$

(d) $\lim_{x \to 4} \sqrt{x} = \sqrt{4} = 2$ (e) $\lim_{x \to k} x^6 = k^6$ (f) $\lim_{x \to 0} \pi = \pi$

3. (a) $\lim_{x \to 1} (3x - 7) = 3 \lim_{x \to 1} x - \lim_{x \to 1} 7 = 3 - 7 = -4$ (Properties 2, 3)

(b) $\lim_{x \to -1} (2x^2 - 5x + 3) = 2 \lim_{x \to -1} x^2 - 5 \lim_{x \to -1} x + \lim_{x \to -1} 3 = 2(-1)^2 - 5(-1) + 3 = 10$
(Properties 2, 1, 3, 6)

(c) $\lim_{x \to 2} (x^3 + x^2 - 2x - 8) = \lim_{x \to 2} x^3 + \lim_{x \to 2} x^2 - 2 \lim_{x \to 2} x - 8 = (2)^3 + (2)^2 - 2(2) - 8 = 0$
(Properties 1, 2, 3, 6)

(d) $\lim_{x \to -2} (x^2 + 5x + 3)^6 = \left[\lim_{x \to -2} x^2 + 5 \lim_{x \to -2} x + \lim_{x \to -2} 3\right]^6 = \left[(-2)^2 + 5(-2) + 3\right]^6$
$= 729$ (Properties 1, 3, 6)

(e) $\lim_{x \to 0} \frac{x-1}{x+1} = \frac{\lim_{x \to 0} x - \lim_{x \to 0} 1}{\lim_{x \to 0} x + \lim_{x \to 0} 1} = \frac{0-1}{0+1} = -1$ (Properties 5, 2, 1)

(f) $\lim_{x \to 4} \frac{x^2 + 2x - 3}{x^2 + 2} = \frac{\lim_{x \to 4} x^2 + 2 \lim_{x \to 4} x - \lim_{x \to 4} 3}{\lim_{x \to 4} x^2 + \lim_{x \to 4} 2} = \frac{4^2 + 2(4) - 3}{4^2 + 2} = \frac{7}{6}$

(Prop's 5, 1, 2, 3, 6)

(g) $\lim_{t \to 2} \frac{t^4 - 3t + 1}{t^2(t-1)^3} = \frac{\lim_{t \to 2} t^4 - 3 \lim_{t \to 2} t + \lim_{t \to 2} 1}{\lim_{t \to 2} t^2 \left[\lim_{t \to 2} t - \lim_{t \to 2} 1\right]^3} = \frac{2^4 - 3(2) + 1}{2^2(2-1)^3} = \frac{11}{4}$

(Properties 5, 2, 3, 1, 4, 6)

(h) $\lim_{u \to -4} \sqrt{u^4 + 2u^2} = \sqrt{\lim_{u \to -4} u^4 + 2 \lim_{u \to -4} u^2} = \sqrt{(-4)^4 + 2(-4)^2} = 12\sqrt{2}$

(Propertiess 7, 1, 3, 6)

Exercise 1.2

(i) $\lim_{x\to 5}\sqrt{x^2+2x-8} = \sqrt{\lim_{x\to 5} x^2 + 2\lim_{x\to 5} x - \lim_{x\to 5} 8} = \sqrt{5^2+2(5)-8} = 3$

(Properties 7, 1, 2, 3, 6)

(j) $\lim_{t\to 3}\left(2t^2+\sqrt{\frac{6+t}{4-t}}\right) = 2\lim_{t\to 3} t^2 + \sqrt{\frac{\lim_{t\to 3} 6 + \lim_{t\to 3} t}{\lim_{t\to 3} 4 - \lim_{t\to 3} t}} = 2(3)^2 + \sqrt{\frac{6+3}{4-3}} = 21$

(Properties 1, 3, 7, 5, 2, 6)

4. (a) $\lim_{x\to -2}\frac{x+2}{x^2-4} = \lim_{x\to -2}\frac{x+2}{(x+2)(x-2)} = \lim_{x\to -2}\frac{1}{x-2} = \frac{1}{-2-2} = -\frac{1}{4}$

(b) $\lim_{x\to 1}\frac{x^2-3x+2}{x-1} = \lim_{x\to 1}\frac{(x-2)(x-1)}{(x-1)} = \lim_{x\to 1}(x-2) = 1-2 = -1$

(c) $\lim_{x\to 3}\frac{x^2-2x-3}{x^2-4x+3} = \lim_{x\to 3}\frac{(x-3)(x+1)}{(x-3)(x-1)} = \lim_{x\to 3}\frac{x+1}{x-1} = 2$

(d) $\lim_{x\to -2}\frac{2x^2+5x+2}{x^2-2x-8} = \lim_{x\to -2}\frac{(2x+1)(x+2)}{(x+2)(x-4)} = \lim_{x\to -2}\frac{2x+1}{x-4} = \frac{2(-2)+1}{-2-4} = \frac{1}{2}$

(e) $\lim_{x\to 1}\frac{x^3-1}{x^2-1} = \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x+1)(x-1)} = \lim_{x\to 1}\frac{x^2+x+1}{x+1} = \frac{1^2+1+1}{1+1} = \frac{3}{2}$

(f) $\lim_{x\to -3}\frac{x+3}{x^3+27} = \lim_{x\to -3}\frac{x+3}{(x+3)(x^2-3x+9)} = \lim_{x\to -3}\frac{1}{x^2-3x+9} = \frac{1}{(-3)^2-3(-3)+9}$

$= \frac{1}{27}$

(g) $\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3} = \lim_{x\to 9}\frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \lim_{x\to 9}(\sqrt{x}+3) = \sqrt{9}+3 = 6.$

(h) $\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2} = \lim_{x\to 2}\frac{\frac{2-x}{2x}}{x-2} = \lim_{x\to 2} -\frac{1}{2x} = -\frac{1}{2(2)} = -\frac{1}{4}$

5. (a) $\lim_{h\to 0}\frac{(4+h)^3-64}{h} = \lim_{h\to 0}\frac{h^3+12h^2+48h+64-64}{h} = \lim_{h\to 0}(h^2+12h+48) =$
$= 0+0+48 = 48$

(b) $\lim_{h\to 0}\frac{(h-2)^2-4}{h} = \lim_{h\to 0}\frac{h^2-4h+4-4}{h} = \lim_{h\to 0}(h-4) = 0-4 = -4$

(c) $\lim_{h\to 0}\frac{\frac{1}{1+h}-1}{h} = \lim_{h\to 0}\frac{\frac{1-1-h}{1+h}}{h} = \lim_{h\to 0}\frac{\frac{-h}{1+h}}{h} = \lim_{h\to 0}\frac{-1}{1+h} = \frac{-1}{1+0} = -1$

(d) $\lim_{h\to 0}\frac{(2+h)^4-16}{h} = \lim_{h\to 0}\frac{h^4+8h^3+24h^2+32h+16-16}{h}$

$= \lim_{h\to 0}(h^3+8h^2+24h+32) = 0+0+0+32 = 32$

(e) $\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h} = \lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}\times\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3} = \lim_{h\to 0}\frac{9+h-9}{h\sqrt{9+h}+3h}$

$= \lim_{h\to 0}\frac{h}{h\sqrt{9+h}+3h} = \lim_{h\to 0}\frac{1}{\sqrt{9+h}+3} = \frac{1}{\sqrt{9+0}+3} = \frac{1}{6}$

(f) $\lim_{h\to 0}\frac{\frac{1}{(2+h)^2}-\frac{1}{4}}{h} = \lim_{h\to 0}\frac{\frac{1}{4+4h+h^2}-\frac{1}{4}}{h} = \lim_{h\to 0}\frac{\frac{4-4-4h-h^2}{16+16h+4h^2}}{h}$

$= \lim_{h\to 0}\frac{\frac{-4h-h^2}{16+16h+4h^2}}{h} = \lim_{h\to 0}\frac{-4-h}{16+16h+4h^2} = \frac{-4-0}{16+0+0} = -\frac{1}{4}$

6. (a) $\lim_{x\to 3}\frac{1}{(x-3)^2}$ does not exist.

(b) $\lim_{x\to -8}\frac{x^2+16x+64}{x+8} = \lim_{x\to -8}\frac{(x+8)^2}{x+8} = \lim_{x\to -8}(x+8) = -8+8=0$

(c) $\lim_{x\to 1}\frac{x^4-1}{x-1} = \lim_{x\to 1}\frac{(x^2+1)(x+1)(x-1)}{x-1} = \lim_{x\to 1}(x^2+1)(x+1) = (1^2+1)(1+1)$

$= 4$

(d) $\lim_{x\to -1}\frac{x-1}{x^2-1}$ does not exist.

(e) $\lim_{x\to 1}\frac{x^2+x-2}{x^2-2x+1} = \lim_{x\to 1}\frac{(x+2)(x-1)}{(x-1)(x-1)} = \lim_{x\to 1}\frac{(x+2)}{(x-1)}$ does not exist.

(f) $\lim_{x\to -2}\frac{x^2-x-2}{x^2+3x+2} = \lim_{x\to -2}\frac{(x-2)(x+1)}{(x+1)(x+2)} = \lim_{x\to -2}\frac{x-2}{x+2}$ does not exist.

(g) $\lim_{x\to 3}\frac{x^{-2}-3^{-2}}{x-3} = \lim_{x\to 3}\frac{\frac{1}{x^2}-\frac{1}{3^2}}{x-3} = \lim_{x\to 3}\frac{\frac{9-x^2}{9x^2}}{x-3} = \lim_{x\to 3}\frac{\frac{-(x+3)(x-3)}{9x^2}}{x-3}$

$= \lim_{x\to 3}\frac{-(x+3)}{9x^2} = \frac{-(3+3)}{9(3)^2} = -\frac{2}{27}$

Exercise 1.2

(h) $\lim_{x\to 4} \dfrac{\frac{1}{\sqrt{x}}-\frac{1}{2}}{x-4} = \lim_{x\to 4} \dfrac{\frac{2-\sqrt{x}}{2\sqrt{x}}}{(\sqrt{x}-2)(\sqrt{x}+2)} = \lim_{x\to 4} \dfrac{-1}{2\sqrt{x}(\sqrt{x}+2)} = \dfrac{-1}{4(\sqrt{4}+2)} = -\dfrac{1}{16}$

(i) $\lim_{x\to 1} \dfrac{x^3-1}{x^3-x^2-4x+4} = \lim_{x\to 1} \dfrac{(x-1)(x^2+x+1)}{(x^2-4)(x-1)} = \lim_{x\to 1} \dfrac{x^2+x+1}{x^2-4} = \dfrac{1+1+1}{1-4}$

$= -1$

(j) $\lim_{x\to 1} \dfrac{x-1}{\sqrt{x}-x} = \lim_{x\to 1} \dfrac{x-1}{\sqrt{x}-x} \times \dfrac{\sqrt{x}+x}{\sqrt{x}+x} = \lim_{x\to 1} \dfrac{(x-1)(\sqrt{x}+x)}{x-x^2} = \lim_{x\to 1} \dfrac{(x-1)(\sqrt{x}+x)}{-x(x-1)}$

$= \lim_{x\to 1} \dfrac{\sqrt{x}+x}{-x} = \dfrac{1+1}{-1} = -2$

7. (a) $f(x) = (1+x)^{\frac{1}{x}}$

$f(1) = 2.000\,000$	$f(0.1) = 2.593\,742$	$f(0.01) = 2.704\,814$
$f(0.001) = 2.716\,924$	$f(0.0001) = 2.718\,146$	$f(0.000\,01) = 2.718\,268$
$f(0.000\,001) = 2.718\,280$	$f(0.000\,000\,1) = 2.718\,282$	

(b) So $\lim_{x\to 0} (1+x)^{\frac{1}{x}} \doteq 2.718\,28$ to five decimal places.

8. (a) $g(x) = \dfrac{2^x-1}{x}$

$g(1) = 1.0000$	$g(0.1) = 0.7177$	$g(0.01) = 0.6956$
$g(0.001) = 0.6934$	$g(0.0001) = 0.6932$	

(b) So $\lim_{x\to 0} \dfrac{2^x-1}{x} \doteq 0.693$ to three decimal places.

9. (a) $\lim_{x\to 8} \dfrac{x-8}{\sqrt[3]{x}-2} = \lim_{x\to 8} \dfrac{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}{\sqrt[3]{x}-2} = \lim_{x\to 8} (\sqrt[3]{x^2}+2\sqrt[3]{x}+4)$

$= \sqrt[3]{8^2}+2\sqrt[3]{8}+4 = 12$

(b) $\lim_{x\to 2} \dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1} = \lim_{x\to 2} \dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1} \times \dfrac{\sqrt{6-x}+2}{\sqrt{6-x}+2} \times \dfrac{\sqrt{3-x}+1}{\sqrt{3-x}+1}$

$= \lim_{x\to 2} \dfrac{(6-x)-4}{(3-x)-1} \times \dfrac{\sqrt{3-x}+1}{\sqrt{6-x}+2} = \lim_{x\to 2} \dfrac{2-x}{2-x} \times \dfrac{\sqrt{3-x}+1}{\sqrt{6-x}+2} = \lim_{x\to 2} \dfrac{\sqrt{3-x}+1}{\sqrt{6-x}+2}$

$= \dfrac{\sqrt{3-2}+1}{\sqrt{6-2}+2} = \dfrac{2}{4} = \dfrac{1}{2}$

10. If $|x-2|<0.005$, then $|f(x)-7| = |(2x+3)-7| = |2x-4| = |2(x-2)|$
$= 2|x-2| < 2(0.005) = 0.01$

11. How close to 1 must x be so $\frac{16x^2-1}{4x-1}$ is within 0.001 of 5?
$\frac{16x^2-1}{4x-1} = \frac{(4x+1)(4x-1)}{4x-1} = 4x+1$. So we want $|(4x+1)-5| < 0.001 \Rightarrow |4x-4| <$
$0.001 \Leftrightarrow 4|x-1| < 0.001 \Leftrightarrow |x-1| < 0.00025$. So x must be within 0.00025 of 1.

12. Show $\lim_{x\to 0} \frac{|x|}{x}$ does not exist.
If $x>0$ then $\frac{|x|}{x} = 1$ since $|x|=x$. So $\frac{|x|}{x}$ approaches 1 as x approaches 0 from above. If $x<0$ then $\frac{|x|}{x} = -1$ since $|x|=-x$. So $\frac{|x|}{x}$ approaches -1 as x approaches 0 from the left. Thus the limit doesn't exist.

13. Find functions f and g such that $\lim_{x\to 0} [f(x)+g(x)]$ exists but $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist.
Take $f(x) = \frac{1}{x}$ and $g(x) = -\frac{1}{x}$. Clearly $\lim_{x\to 0} \frac{1}{x}$ and $\lim_{x\to 0} -\frac{1}{x}$ don't exist, but
$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x}\right) = \lim_{x\to 0} 0 = 0$ exists.

Exercise 1.3

1. From the graph.

(a) $\lim_{x \to -2^+} f(x) = 0$ (b) $\lim_{x \to 0^-} f(x) = 2$

(c) $\lim_{x \to 0^+} f(x) = 1$ (d) $\lim_{x \to 0} f(x)$ does not exist (e) $\lim_{x \to 2^-} f(x) = 3$

(f) $\lim_{x \to 2^+} f(x) = 3$ (g) $\lim_{x \to 2} f(x) = 3$ (h) $\lim_{x \to 4^-} f(x) = 4$

2. From the graph.

(a) $\lim_{x \to -3^+} g(x) = 2$ (b) $\lim_{x \to -1^-} g(x) = 2$

(c) $\lim_{x \to -1^+} g(x) = 1$ (d) $\lim_{x \to -1} g(x)$ does not exist (e) $\lim_{x \to 2^-} g(x) = 0$

(f) $\lim_{x \to 2^+} g(x) = 0$ (g) $\lim_{x \to 2} g(x) = 0$ (h) $\lim_{x \to 1} g(x) = 1$

3. See graph

(a) $x = -2$, continuous (b) $x = 0$, discontinuous

(c) $x = 2$, discontinuous (d) $x = 4$, continuous (e) $x = 6$, discontinuous

4. (a) $\lim_{x \to 0^+} \sqrt[4]{x} = \sqrt[4]{\lim_{x \to 0^+} x} = \sqrt[4]{0} = 0$ (Property 7)

(b) $\lim_{x \to 3^+} \sqrt{x-3} = \sqrt{\lim_{x \to 3^+} x - \lim_{x \to 3^+} 3} = \sqrt{3-3} = 0$ (Properties 7,2)

(c) $\lim_{x \to 1^-} \sqrt{1-x} = \sqrt{\lim_{x \to 1^-} 1 - \lim_{x \to 1^-} x} = \sqrt{1-1} = 0$ (Properties 7,2)

(d) $\lim_{x \to \frac{1}{2}^-} \sqrt[4]{1-2x} = \sqrt[4]{\lim_{x \to \frac{1}{2}^-} 1 - 2 \lim_{x \to \frac{1}{2}^-} x} = \sqrt{1-1} = 0$ (Properties 7,2,3)

(e) $\lim_{x \to 6^+} |x-6| = \left| \lim_{x \to 6^+} x - \lim_{x \to 6^+} 6 \right| = |6-6| = 0$ (Property 2)

(f) $\lim_{x \to 6^-} |x-6| = \left| \lim_{x \to 6^-} x - \lim_{x \to 6^-} 6 \right| = |6-6| = 0$ (Property 2)

(g) $\lim_{x \to 6} |x-6| = 0$ (Since the above two limits are 0)

(h) $\lim_{x \to 0^+} \frac{|x|}{x} = \frac{\left| \lim_{x \to 0^+} x \right|}{\lim_{x \to 0^+} x} = 1$ (Property 5)

Exercise 1.3

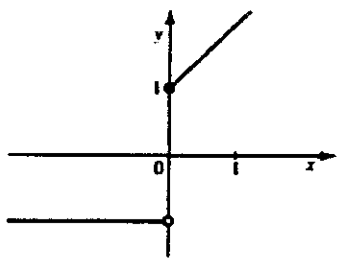
(i) $\lim_{x \to 0^-} \frac{|x|}{x} = \frac{\left|\lim_{x \to 0^-} x\right|}{\lim_{x \to 0^-} x} = -1$ (Property 5)

(j) $\lim_{x \to 0} \frac{|x|}{x}$ does not exist

5. (a) Since $f(x) = -1$ for $x < 0$, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} -1 = -1$

(b) Since $f(x) = x + 1$ for $x \geq 0$, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x+1) = 0 + 1 = 1$

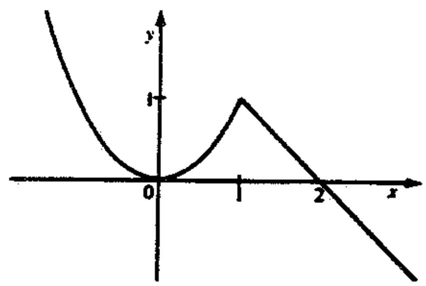
(c) $\lim_{x \to 0} f(x)$ does not exist



6. (a) Since $g(x) = x^2$ for $x \leq 1$, $\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} x^2 = (1)^2 = 1$

(b) Since $g(x) = 2 - x$ for $x > 1$, $\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} (2-x) = 2 - 1 = 1$

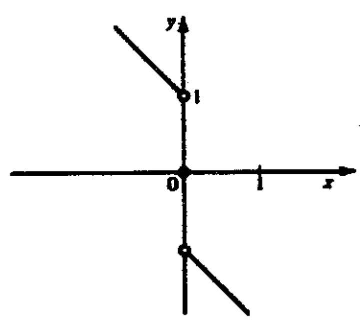
(c) $\lim_{x \to 1} g(x) = 1$



7. (a) Since $h(x) = (1 - x)$ for $x < 0$, $\lim_{x \to 0^-} h(x) = \lim_{x \to 0^-} (1-x) = 1 - 0 = 1$

(b) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (-x-1) = 0 - 1 = -1$

(c) $\lim_{x \to 1} h(x)$ does not exist



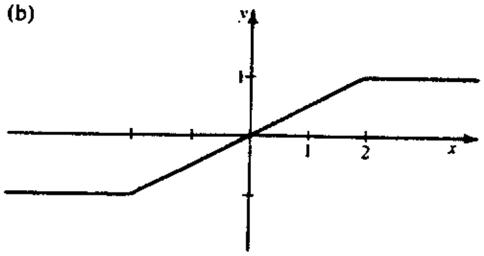
8. (a) i) Since $f(x) = -1$ for $x \le -2$, $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} -1 = -1$

(ii) $\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{1}{2}x = \frac{1}{2}(-2) = -1$

(iii) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{1}{2}x = \frac{1}{2}(2) = 1$

(iv) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 1 = 1$

(b)



(c) f is continuous everywhere.

9. (a) (i) Since $f(x) = (x+1)^2$ for $x < -1$, $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (x+1)^2 = (-1+1)^2 = 0$

(ii) $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x = -1$

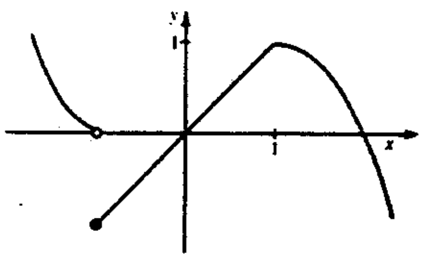
(iii) $\lim_{x \to -1} f(x)$ does not exist

(iv) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1$

(v) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x - x^2) = (2(1) - (1)^2) = 1$

(vi) $\lim_{x \to 1} f(x) = 1$

(b)



(c) f is discontinuous at $x = -1$.

10. (a) Only possible discontinuity could occur at $x = 4$. Since $f(x) = 2x + 3$ for $x \neq 4$,

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} (2x+3) = 2(4) + 3 = 11$ and $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} (2x+3) = 2(4) + 3 = 11$.

Since $f(4) = 12$, f is discontinuous at $x = 4$.

(b) Only possible discontinuities are at $x = 0$, $x = 1$. Since $\lim_{x \to 0^+} f(x) = 1$ and $\lim_{x \to 0^-} f(x) = 1$ and $f(0) = 1$, there is no discontinuity at $x = 0$.

Since $\lim_{x \to 1^+} f(x) = 0$ and $\lim_{x \to 1^-} f(x) = 2$ there is a discontinuity at $x = 1$.

(c) Only possible discontinuities are at $x = -1$, $x = 1$. Since $\lim_{x \to -1^+} f(x) = -1$ and $\lim_{x \to -1^-} f(x) = 1$, f is discontinuous at $x = -1$. Since $\lim_{x \to 1^+} f(x) = 1$ and $\lim_{x \to 1^-} f(x) = 1$ and $f(1) = 1$, f is continuous at $x = 1$.

(d) Only possible discontinuities are at $x = 1$, $x = 3$. Since $\lim_{x \to 1^+} f(x) = -1$ and $\lim_{x \to 1^-} f(x) = 1$, f is discontinuous at $x = 1$. Since $\lim_{x \to 3^+} f(x) = -1$ and $\lim_{x \to 3^-} f(x) = 1$, there is also a discontinuity at $x = 3$.

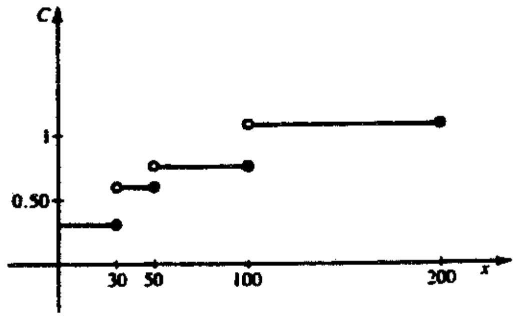
11. $C(x) = 0.38$ if $x \leq 30$

$C(x) = 0.59$ if $30 < x \leq 50$

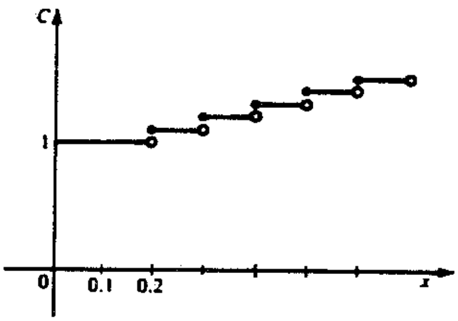
$C(x) = 0.76$ if $50 < x \leq 100$

$C(x) = 1.14$ if $100 < x \leq 200$

C is discontinuous at 0, 30, 50, and 100.

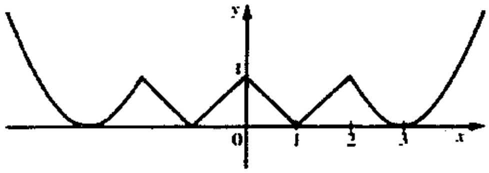


12. Discontinuities at $x = 0, 0.2, 0.3, 0.4, 0.5 \ldots$



Exercise 1.3

13. From the graph, f is continuous on R.



14. f(x) is continuous at every number if $x+c = cx^2+1$ when $x = 2$.
So $2+c = c(2)^2+1 \Rightarrow 3c = 1 \Rightarrow c = \frac{1}{3}$.

Exercise 1.4

1. $y = 2x - x^2$, $(2,0)$ (a) (i) $m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{2x - x^2 - (2(2) - (2)^2)}{x - 2}$

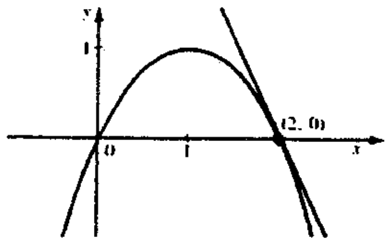
$= \lim_{x \to 2} \frac{2x - x^2 - (0)}{x - 2} = \lim_{x \to 2} \frac{-x(x-2)}{x-2} = \lim_{x \to 2} -x = -2$

(ii) $m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{2(2+h) - (2+h)^2 - 2(2) + 2^2}{h}$

$= \lim_{h \to 0} \frac{4 + 2h - 4 - 4h - h^2}{h} = \lim_{h \to 0} \frac{-h^2 - 2h}{h} = \lim_{h \to 0} (-h - 2) = -2$

(b) $y - 0 = -2(x - 2) \Rightarrow y = -2x + 4$ or $2x + y - 4 = 0$

(c)



2. $y = x^3$, $(1,1)$ (a) (i) $m = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{x^3 - (1)^3}{x - 1}$

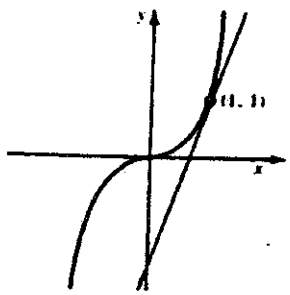
$= \lim_{x \to 1} \frac{(x-1)(x^2 + x + 1)}{x - 1} = \lim_{x \to 1} (x^2 + x + 1) = 1^2 + 1 + 1 = 3$

(ii) $m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)^3 - (1)^3}{h} = \lim_{h \to 0} \frac{h^3 + 3h^2 + 3h + 1 - 1}{h}$

$= \lim_{h \to 0} (h^2 + 3h + 3) = 3$

(b) $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$ or $3x - y - 2 = 0$

(c)



3. $y = 2x^2 + 4x - 1$, $(2,15)$

$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{2(2+h)^2 + 4(2+h) - 1 - [2(2)^2 + 4(2) - 1]}{h}$

$= \lim_{h \to 0} \frac{2(2)^2 + 8h + 2h^2 + 8 + 4h - 1 - 8 - 8 + 1}{h} = \lim_{h \to 0} \frac{8h + 2h^2 + 4h}{h}$

$= \lim_{h \to 0} (12 + 2h) = 12$

Exercise 1.4

4. $xy = 1 \Rightarrow y = \frac{1}{x}$, $(-2, -\frac{1}{2})$

$$m = \lim_{x \to -2} \frac{f(x) - f(-2)}{x+2} = \lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} = \lim_{x \to -2} \frac{\frac{2+x}{2x}}{x+2} = \lim_{x \to -2} \frac{1}{2x} = \frac{1}{2(-2)} = -\frac{1}{4}$$

5. $y = \sqrt{x-2}$, $(6,2)$

$$m = \lim_{x \to 6} \frac{f(x) - f(6)}{x-6} = \lim_{x \to 6} \frac{\sqrt{x-2} - \sqrt{6-2}}{x-6} = \lim_{x \to 6} \frac{\sqrt{x-2} - 2}{x-6}$$

$$= \lim_{x \to 6} \frac{\sqrt{x-2} - 2}{x-6} \times \frac{\sqrt{x-2} + 2}{\sqrt{x-2} + 2} = \lim_{x \to 6} \frac{x-2-4}{(x-6)(\sqrt{x-2} - 2)} = \lim_{x \to 6} \frac{x-6}{(x-6)(\sqrt{x-2} + 2)}$$

$$= \lim_{x \to 6} \frac{1}{\sqrt{x-2}+2} = \frac{1}{\sqrt{6-2}+2} = \frac{1}{4}$$

6. $y = x^2 + 4x - 1$

(a) (i) $x = -3$, $m = \lim_{h \to 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h \to 0} \frac{(-3+h)^2 + 4(-3+h) - 1 - (-4)}{h}$

$$= \lim_{h \to 0} \frac{9 - 6h + h^2 - 12 + 4h + 3}{h} = \lim_{h \to 0} \frac{-2h + h^2}{h} = \lim_{h \to 0} (h-2) = -2$$

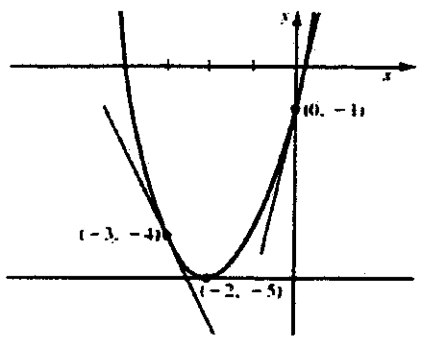
(ii) $x = -2$, $m = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h \to 0} \frac{(-2+h)^2 + 4(-2+h) - 1 - (-5)}{h}$

$$= \lim_{h \to 0} \frac{4 - 4h + h^2 - 8 + 4h + 4}{h} = \lim_{h \to 0} \frac{h^2}{h} = \lim_{h \to 0} (h) = 0$$

(iii) $x = 0$, $m = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{h^2 + 4h - 1 - (-1)}{h} = \lim_{h \to 0} \frac{h^2 + 4h}{h}$

$$= \lim_{h \to 0} (h+4) = 4$$

(b)



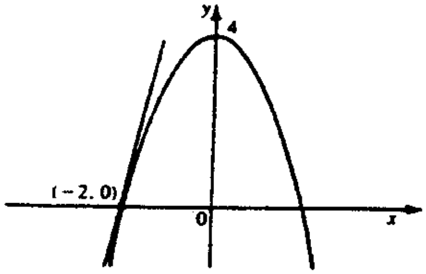
7. (i) $y = 4 - x^2$, $(-2,0)$

(a) $m = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h \to 0} \frac{4 - (-2+h)^2 - [4 - (-2)^2]}{h}$

$$= \lim_{h \to 0} \frac{4 - 4 + 4h - h^2 - 4 + 4}{h} = \lim_{h \to 0} \frac{4h - h^2}{h} = \lim_{h \to 0} (4-h) = 4$$

(b) $y-0=4(x+2) \Rightarrow y=4x+8$ or $4x-y+8=0$

(c)



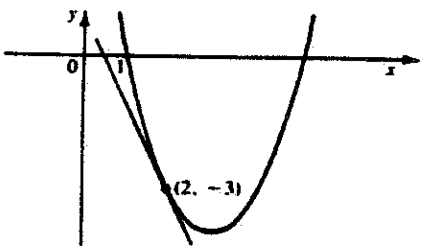
(ii) $y=x^2-6x+5$, $(2,-3)$

(a) $m=\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\dfrac{(2+h)^2-6(2+h)+5-[(2)^2-6(2)+5]}{h}$

$=\lim_{h\to 0}\dfrac{4+4h+h^2-12-6h+5+3}{h}=\lim_{h\to 0}\dfrac{h^2-2h}{h}=\lim_{h\to 0}(h-2)=-2$

(b) $y+3=-2(x-2) \Rightarrow y=-2x+1$ or $2x+y-1=0$

(c)

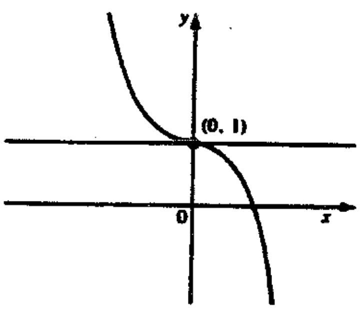


(iii) $y=1-x^3$, $(0,1)$

(a) $m=\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\dfrac{1-h^3-1}{h}=\lim_{h\to 0}(-h^2)=0$

(b) $y-1=0(x-0) \Rightarrow y=1$

(c)



Exercise 1.4

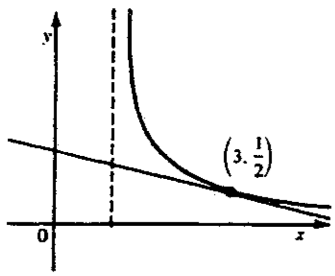
(iv) $y=\frac{1}{x-1}$, $(3,\frac{1}{2})$

(a) $m=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{\frac{1}{3+h-1}-\frac{1}{3-1}}{h}=\lim_{h\to 0}\frac{\frac{2-2-h}{2(2+h)}}{h}=\lim_{h\to 0}\frac{-1}{2(2+h)}$

$=-\frac{1}{4}$

(b) $y-\frac{1}{2}=-\frac{1}{4}(x-3)\Rightarrow y=-\frac{1}{4}x+\frac{5}{4}$ or $x+4y-5=0$

(c)



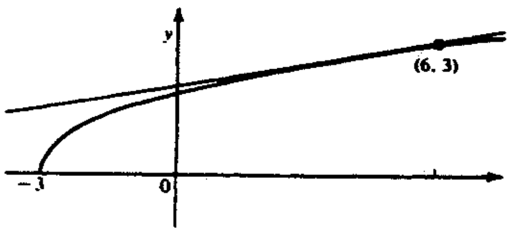
(v) $y=\sqrt{x+3}$, $(6,3)$

(a) $m=\lim_{h\to 0}\frac{f(6+h)-f(6)}{h}=\lim_{h\to 0}\frac{\sqrt{6+h+3}-\sqrt{6+3}}{h}=\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}$

$=\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}\times\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}=\lim_{h\to 0}\frac{9+h-9}{h\sqrt{9+h}+3h}=\lim_{h\to 0}\frac{1}{\sqrt{9+h}+3}=\frac{1}{6}$

(b) $y-3=\frac{1}{6}(x-6)\Rightarrow y=\frac{1}{6}x+2$ or $x-6y+12=0$

(c)



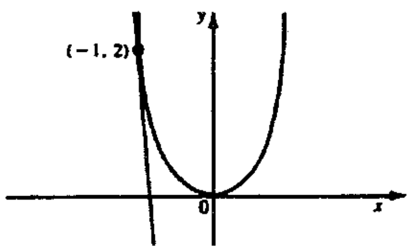
(vi) $y=2x^4$, $(-1,2)$

(a) $m=\lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h}=\lim_{h\to 0}\frac{2(-1+h)^4-2(-1)^4}{h}=\lim_{h\to 0}\frac{2(-1+h)^4-2(-1)^4}{h}$

$=\lim_{h\to 0}\frac{2h^4-8h^3+12h^2-8h+2-2}{h}=\lim_{h\to 0}(2h^3-8h^2+12h-8)=-8$

(b) $y-2=-8(x+1)\Rightarrow y=-8x-6$ or $8x+y+6=0$

(c)



Exercise 1.4

8. (a) $f(x) = 4 - x + 3x^2$, $(-1, 8)$;

$$m = \lim_{h \to 0} \frac{4 - (-1+h) + 3(-1+h)^2 - (8)}{h} = \lim_{h \to 0} \frac{3h^2 - 7h}{h} = \lim_{h \to 0} (3h - 7) = -7.$$

So $y - 8 = -7(x+1) \Rightarrow y = -7x + 1$ or $7x + y - 1 = 0$.

(b) $f(x) = x^3 - x$, $(0,0)$;

$$m = \lim_{h \to 0} \frac{(0+h)^3 - (0+h) - 0}{h} = \lim_{h \to 0} \frac{h^3 - h}{h} = \lim_{h \to 0} (h^2 - 1) = -1$$

So $y - 0 = -1(x - 0) \Rightarrow y = -x$ or $x + y = 0$.

(c) $g(x) = \frac{2x+1}{x-1}$, $(2,5)$;

$$m = \lim_{h \to 0} \frac{\frac{2(2+h)+1}{(2+h)-1} - \frac{2(2)+1}{2-1}}{h} = \lim_{h \to 0} \frac{\frac{2h+5}{h+1} - 5}{h} = \lim_{h \to 0} \frac{\frac{2h+5-5h-5}{h+1}}{h}$$

$$= \lim_{h \to 0} \frac{-3}{h+1} = -3.$$ So $y - 5 = -3(x-2) \Rightarrow y = -3x + 11$ or $3x + y - 11 = 0$.

(d) $g(x) = \frac{1}{\sqrt{x}}$, $(1,1)$;

$$m = \lim_{h \to 0} \frac{\frac{1}{\sqrt{1+h}} - \frac{1}{\sqrt{1}}}{h} = \lim_{h \to 0} \frac{\frac{1-\sqrt{1+h}}{\sqrt{1+h}}}{h} = \lim_{h \to 0} \frac{\frac{1-\sqrt{1+h}}{\sqrt{1+h}}}{h} \times \frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}$$

$$= \lim_{h \to 0} \frac{\frac{1-1-h}{\sqrt{1+h}}}{h + h\sqrt{1+h}} = \lim_{h \to 0} \frac{-1}{\sqrt{1+h} + 1 + h} = -\frac{1}{2}$$

So $y - 1 = -\frac{1}{2}(x - 1) \Rightarrow y = -\frac{1}{2}x + \frac{3}{2}$ or $x + 2y - 3 = 0$.

9. (a) $y = x^2 + x + 1$, $x = a$;

$$m = \lim_{h \to 0} \frac{(a+h)^2 + (a+h) + 1 - [a^2 + a + 1]}{h} = \lim_{h \to 0} \frac{a^2 + 2ah + h^2 + a + h + 1 - a^2 - a - 1}{h}$$

$$= \lim_{h \to 0} \frac{2ah + h^2 + h}{h} = \lim_{h \to 0} (2a + h + 1) = 2a + 1$$

(b) Using $m = 2a + 1$. $x = -1$, $m = -1$ $x = -\frac{1}{2}$, $m = 0$

$x = 0$, $m = 1$ $x = \frac{1}{2}$, $m = 2$ $x = 1$, $m = 3$

10. (a) $y = 3x^2 + 2x$, $x = a$; $m = \lim_{h \to 0} \frac{3(a+h)^2 + 2(a+h) - [3a^2 + 2a]}{h}$

$= \lim_{h \to 0} \frac{3a^2 + 6ah + 3h^2 + 2a + 2h - 3a^2 - 2a}{h} = \lim_{h \to 0} \frac{6ah + 3h^2 + 2h}{h}$

$= \lim_{h \to 0} (6a + 3h + 2) = 6a + 2$

(b) At what point on the parabola is the tangent || to $y = 10x - 2$?

|| tangent $\Rightarrow 6a + 2 = 10 \Rightarrow a = \frac{4}{3}$. So the point is $(\frac{4}{3}, 8)$.

11. $y = \frac{1}{2}x^2$, $y = 1 - \frac{1}{2}x^2$. Intersection gives $\frac{1}{2}x^2 = 1 - \frac{1}{2}x^2 \Rightarrow x = \pm 1$.

When $x = \pm 1$, $y = \frac{1}{2}$. For $y = \frac{1}{2}x^2$, $x = a$; $m = \lim_{h \to 0} \frac{\frac{1}{2}(a+h)^2 - \frac{1}{2}a^2}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{2}a^2 + ah + \frac{1}{2}h^2 - \frac{1}{2}a^2}{h} = \lim_{h \to 0} \frac{ah + \frac{1}{2}h^2}{h} = \lim_{h \to 0} (a + \frac{1}{2}h) = a$.

For $y = 1 - \frac{1}{2}x^2$, $x = a$; $m = \lim_{h \to 0} \frac{1 - \frac{1}{2}(a+h)^2 - 1 + \frac{1}{2}a^2}{h} = \lim_{h \to 0} \frac{1 - \frac{1}{2}a^2 - ah + \frac{1}{2}h^2 - 1 + \frac{1}{2}a^2}{h}$

$= \lim_{h \to 0} \frac{-ah + \frac{1}{2}h^2}{h} = \lim_{h \to 0} (-a + \frac{1}{2}h) = -a$.

At $x = 1$. For $y = \frac{1}{2}x^2$, $m = 1$. For $y = 1 - \frac{1}{2}x^2$, $m = -1$. So the tangents are $\perp$.

At $x = -1$. For $y = \frac{1}{2}x^2$, $m = -1$. For $y = 1 - \frac{1}{2}x^2$, $m = 1$. So the tangents are $\perp$.

Exercise 1.5

1. $y = f(t) = 30t - 4.9t^2$, t in seconds.

(a) Average velocity $\bar{v} = \frac{\Delta s}{\Delta t} = \frac{30t - 4.9t^2 - [30(2) - 4.9(2)^2]}{t-2} = \frac{30t - 4.9t^2 - 40.4}{t-2}$ m/s

(i) $t = 3$ s, $\bar{v} = \frac{30(3) - 4.9(3)^2 - 40.4}{3-2} = 5.5$ m/s

(ii) $t = 2.5$ s, $\bar{v} = \frac{30(2.5) - 4.9(2.5)^2 - 40.4}{3-2.5} = 7.95$ m/s

(iii) $t = 2.1$ s, $\bar{v} = \frac{30(2.1) - 4.9(2.1)^2 - 40.4}{3-2.1} = 9.91$ m/s

(iv) $t = 2.05$ s, $\bar{v} = \frac{30(2.05) - 4.9(2.05)^2 - 40.4}{3-2.05} = 10.155$ m/s

(v) $t = 2.01$ s, $\bar{v} = \frac{30(2.01) - 4.9(2.01)^2 - 40.4}{3-2.01} = 10.351$ m/s

(b) ' istantaneous velocity when $t = 2$, $v(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \frac{30(2+h) - 4.9(2+h)^2 - 40.4}{h} = \lim_{h \to 0} \frac{60 + 30h - 4.9h^2 - 19.6h - 19.6 - 40.4}{h}$

$= \lim_{h \to 0} \frac{10.4h - 4.9h^2}{h} = \lim_{h \to 0} (10.4 - 4.9h) = 10.4$ m/s

2. $s = f(t) = t^2 - 4t + 3$. t in sec., s in m.

(a) Average velocity $\bar{v} = \frac{\Delta s}{\Delta t} = \frac{t^2 - 4t + 3 - [3^2 - 4(3) + 3]}{t-3} = \frac{t^2 - 4t + 3}{t-3}$ m/s

(i) $3 \le t \le 5$, $\bar{v} = \frac{5^2 - 4(5) + 3}{5-3} = 4$ m/s

(ii) $3 \le t \le 4$, $\bar{v} = \frac{4^2 - 4(4) + 3}{4-3} = 3$ m/s

(iii) $3 \le t \le 3.5$, $\bar{v} = \frac{(3.5)^2 - 4(3.5) + 3}{3.5-3} = 2.5$ m/s

(iv) $3 \le t \le 3.1$, $\bar{v} = \frac{(3.1)^2 - 4(3.1) + 3}{3.1-3} = 2.1$ m/s

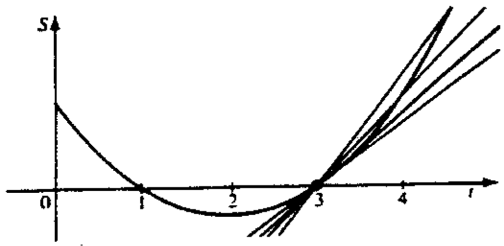
(b) Instantaneous velocity when $t = 3$, $v(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$

$= \lim_{h \to 0} \frac{(3+h)^2 - 4(3+h) + 3 - 0}{h} = \lim_{h \to 0} \frac{9 + 6h + h^2 - 12 - 4h + 3}{h} = \lim_{h \to 0} \frac{2h + h^2}{h}$

$= \lim_{h \to 0} (2+h) = 2$ m/s

Exercise 1.5

(c), (d)



3. $s = f(t) = 2t^2 + 4t - 5$, t in sec., s in m.

At time $t = a$, $v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$= \lim_{h \to 0} \frac{2(a+h)^2 + 4(a+h) - 5 - [2a^2 + 4a - 5]}{h}$

$= \lim_{h \to 0} \frac{2h^2 + 4ah + 4h}{h} = \lim_{h \to 0} (2h + 4a + 4) = 4a + 4.$

$v(1) = 4(1) + 4 = 8$ m/s, $v(2) = 12$ m/s, $v(3) = 16$ m/s.

4. See Section 1.5, Example 3

(a) (i) $3 \leq t \leq 5$, $\frac{\Delta T}{\Delta t} = \frac{T(5) - T(3)}{5 - 3} = \frac{5.3 - 6.5}{2} = -0.6$ °/min

(ii) $3 \leq t \leq 4$, $\frac{\Delta T}{\Delta t} = \frac{T(4) - T(3)}{4 - 3} = \frac{5.7 - 6.5}{1} = -0.8$ °/min

(iii) $1 \leq t \leq 3$, $\frac{\Delta T}{\Delta t} = \frac{T(3) - T(1)}{3 - 1} = \frac{6.5 - 12}{2} = -2.75$ °/min

(iv) $2 \leq t \leq 3$, $\frac{\Delta T}{\Delta t} = \frac{T(3) - T(2)}{3 - 2} = \frac{6.5 - 8.3}{1} = -1.8$ °/min

(b) At $t = 3$, $\frac{\Delta T}{\Delta t} = -1$ °/min

5. Average rate of growth $\frac{\Delta P}{\Delta Y} = \frac{P(Y) - 229}{Y - 1984}$ thousand/year

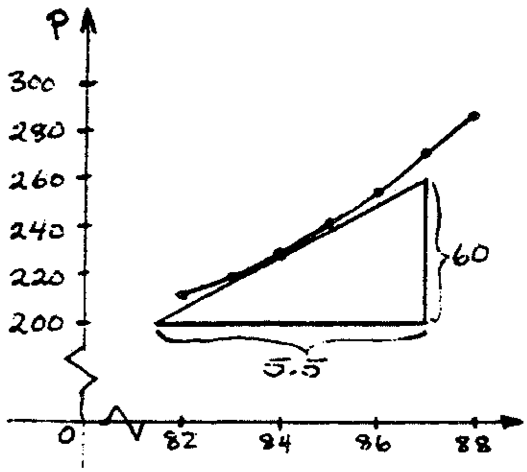
(a) (i) $Y = 1988$, $\frac{\Delta P}{\Delta Y} = \frac{286 - 229}{1988 - 1984} = 14.3$ thousand/year

(ii) $Y = 1987$, $\frac{\Delta P}{\Delta Y} = \frac{270 - 229}{1987 - 1984} = 13.7$ thousand/year

(iii) $Y = 1986$, $\frac{\Delta P}{\Delta Y} = \frac{255 - 229}{1986 - 1984} = 13.0$ thousand/year

(iv) $Y = 1985$, $\frac{\Delta P}{\Delta Y} = \frac{241 - 229}{1985 - 1984} = 12.0$ thousand/year

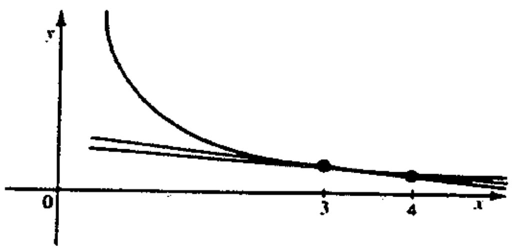
(b) From the graph, the instantaneous rate of growth is estimated to be $\frac{60}{5.5} \doteq 11$ thousand/year.



6. (a) $y = \frac{2}{x}$, $3 \le x \le 4$; Average rate of change, $\frac{\Delta y}{\Delta x} = \frac{\frac{2}{4} - \frac{2}{3}}{4-3} = -\frac{1}{6}$

(b) Instantaneous rate of change when $x = 3$, $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h}$

$$= \lim_{h \to 0} \frac{\frac{6 - 2(3+h)}{3(3+h)}}{h} = \lim_{h \to 0} \frac{\frac{6-6-2h}{9+3h}}{h} = \lim_{h \to 0} \frac{-2}{9+3h} = -\frac{2}{9}$$



7. (a) $V = x^3$, $\frac{\Delta V}{\Delta x} = \frac{x^3 - 4^3}{x-4} = \frac{x^3 - 64}{x-4}$

(i) When $x = 5$, $\frac{\Delta V}{\Delta x} = \frac{5^3 - 64}{5-4} = 61$ mm³/mm

(ii) When $x = 4.1$, $\frac{\Delta V}{\Delta x} = \frac{(4.1)^3 - 64}{4.1 - 4} = 49.21$ mm³/mm

(iii) When $x = 4.01$, $\frac{\Delta V}{\Delta x} = \frac{(4.01)^3 - 64}{4.01 - 4} = 48.1201$ mm³/mm

(b) At $x = 4$, $\lim_{h \to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h \to 0} \frac{(4+h)^3 - 4^3}{h}$

$$= \lim_{h \to 0} \frac{h^3 + 12h^2 + 48h + 64 - 64}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 12h^2 + 48h}{h} = \lim_{h \to 0} (h^2 + 12h + 48) = 48 \text{ mm}^3/\text{mm}$$

8. $V = 1000\left(1-\frac{t}{60}\right)^2,\ 0 \le t \le 60$

Instantaneous rate of change of V with respect to t when t = 10 min

$$\lim_{t\to 0}\frac{\Delta V}{\Delta t} = \lim_{t\to 10}\frac{V(t)-V(10)}{t-10} = \lim_{t\to 10}\frac{1000\left(1-\frac{t}{60}\right)^2 - 1000\left(1-\frac{10}{60}\right)^2}{t-10}$$

$$= \lim_{t\to 10}\frac{1000\left(1-\frac{t}{30}+\frac{t^2}{3600}\right)-\frac{25\,000}{36}}{t-10} = \lim_{t\to 10}\frac{\frac{36\,000}{36}-\frac{25\,000}{36}-\frac{100t}{3}+\frac{10t^2}{36}}{t-10}$$

$$= \lim_{t\to 10}\frac{\frac{10t^2-1200t+11\,000}{36}}{t-10} = \lim_{t\to 10}\frac{\frac{10(t-10)(t-110)}{36}}{t-10} = \lim_{t\to 10}\frac{10(t-110)}{36}$$

$= -\frac{1000}{36} = -\frac{250}{9}$ L/min. So the water is leaving the tank at a rate of $\frac{250}{9}$ L/min.

9. $s = f(t) = 50t - 0.83t^2$

(a) Average velocity from t = 1 s, $\bar{v} = \frac{f(t)-f(1)}{t-1} = \frac{50t-0.83t^2-49.17}{t-1}$ m/s

(i) $1 \le t \le 2$, $\bar{v} = \frac{50(2)-0.83(2)^2-49.17}{2-1} = 47.51$ m/s

(ii) $1 \le t \le 1.5$, $\bar{v} = \frac{50(1.5)-0.83(1.5)^2-49.17}{1.5-1} = 47.93$ m/s

(iii) $1 \le t \le 1.1$, $\bar{v} = \frac{50(1.1)-0.83(1.1)^2-49.17}{1.1-1} = 48.26$ m/s

(iv) $1 \le t \le 1.05$, $\bar{v} = \frac{50(1.05)-0.83(1.05)^2-49.17}{1.05-1} = 48.30$ m/s

(v) $1 \le t \le 1.01$, $\bar{v} = \frac{50(1.01)-0.83(1.01)^2-49.17}{1.01-1} = 48.33$ m/s

(b) Instantaneous velocity when t = 1 s, $v = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$$= \lim_{h\to 0}\frac{50(1+h)-0.83(1+h)^2-49.17}{h} = \lim_{h\to 0}\frac{50+50h-0.83-1.66h-0.83h^2-49.17}{h}$$

$$= \lim_{h\to 0}\frac{48.34h-0.83h^2}{h} = \lim_{h\to 0}(48.34-0.83h) = 48.34 \text{ m/s}$$

(c) Velocity after t seconds,

$$v(t) = \lim_{h\to 0}\frac{f(t+h)-f(t)}{h} = \lim_{h\to 0}\frac{50(t+h)-0.83(t+h)^2-50t+0.83t^2}{h}$$

$$= \lim_{h\to 0}\frac{50t+50h-0.83t^2-1.66ht-0.83h^2-50t+0.83t^2}{h} = \lim_{h\to 0}\frac{50h-1.66ht-0.83h^2}{h}$$

$$= \lim_{h\to 0}(50-1.66t-0.83h) = 50-1.66t$$

(d) When the arrow hits the moon, $s(t) = 0$, so $50t - 0.83t^2 = 0 \Rightarrow t(50-0.83t) = 0$

$\Rightarrow t = 0$ or $t = \frac{50}{0.83} \doteq 60.24$ s

(e) $v\left(\frac{50}{0.83}\right) = 50 - 1.66\left(\frac{50}{0.83}\right) = -50$ m/s. So the arrow hits the surface with a velocity of 50 m/s.

Exercise 1.6

1. (a) $t_n = \left(\frac{1}{3}\right)^n$, $\lim_{n\to\infty} t_n = \lim_{n\to\infty} \left(\frac{1}{3}\right)^n = 0$

(b) $t_n = 4 + \frac{1}{n}$, $\lim_{n\to\infty} t_n = \lim_{n\to\infty} (4 + \frac{1}{n}) = 4 + 0 = 4$

(c) $t_n = n$, $\lim_{n\to\infty} t_n$ does not exist

(d) $t_n = 3$, $\lim_{n\to\infty} t_n = 3$

(e) $\lim_{n\to\infty} t_n = 0$ since both the even terms and odd terms approach 0.

(f) $t_n = 6 + \frac{(-1)^n}{n}$, $\lim_{n\to\infty} t_n = \lim_{n\to\infty} \left(6 + \frac{(-1)^n}{n}\right) = 6 + 0 = 6$

(g) $\lim_{n\to\infty} t_n$ does not exist since the odd terms approach 1 while the even terms approach 0.

2. (a) $t_n = \frac{n-1}{2n-1}$, $0, \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}$

(b) $t_n = \frac{2n}{n^2+1}$, $1, \frac{4}{5}, \frac{6}{10}, \frac{8}{17}, \frac{10}{26}, \frac{12}{37}$

(c) $t_n = n2^n$, 2, 8, 24, 64, 160, 384

(d) $t_n = \frac{(-1)^{n-1}}{n}$, $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}$

(e) $t_1 = 1$, $t_n = \frac{1}{1+t_{n-1}}$ $(n \geq 2)$, $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}$

(f) $t_1 = 1$, $t_2 = 2$, $t_n = t_{n-1} - t_{n-2}$ $(n \geq 3)$, $1, 2, 1, -1, -2, -1$

3. (a) $\lim_{n\to\infty} \frac{1}{n^2} = 0$

(b) $\lim_{n\to\infty} \frac{1}{5+n} = 0$ since $5 + n \to \infty$ as $n \to \infty$

(c) $\lim_{n\to\infty} \left(6 + \frac{1}{n^3}\right) = 6 + 0 = 6$

(d) $\lim_{n\to\infty} \frac{n}{3n-1} = \frac{1}{3 - \frac{1}{n}} = \frac{1}{3-0} = \frac{1}{3}$

(e) $\lim_{n\to\infty} \frac{6n+9}{3n-2} = \lim_{n\to\infty} \frac{6 + \frac{9}{n}}{3 - \frac{2}{n}} = \frac{6+0}{3-0} = 2$

(f) $\lim_{n\to\infty} 5n$ does not exist

(g) $\lim_{n\to\infty} \frac{n^2+1}{2n^2-1} = \lim_{n\to\infty} \frac{1 + \frac{1}{n^2}}{2 - \frac{1}{n^2}} = \frac{1+0}{2-0} = \frac{1}{2}$

(h) $\lim_{n\to\infty} \frac{(n+1)^2}{n(n+2)} = \lim_{n\to\infty} \frac{n^2+2n+1}{n^2+2n} = \lim_{n\to\infty} \frac{1 + \frac{2}{n} + \frac{1}{n^2}}{1 + \frac{2}{n}} = \frac{1+0+0}{1+0} = 1$

(i) The terms are $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{6}, -\frac{1}{6}, \ldots$. Thus $\lim_{n\to\infty} \frac{(-1)^{n+1}}{n} = 0$

(j) $\lim_{n\to\infty} \left(-\frac{1}{4}\right)^n = 0$ since $4^n \to \infty$ as $n \to \infty$

Exercise 1.6

(k) $\lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1+\frac{1}{n^2}} = \frac{0}{1+0} = 0$

(l) The terms are 1, −2, 3, −4, 5, −6, … . $\lim_{n\to\infty} (-1)^{n-1}n$ does not exist.

(m) $\lim_{n\to\infty} 5^{-n} = \lim_{n\to\infty} \frac{1}{5^n} = 0$

(n) $\lim_{n\to\infty} (n^3+n^2)$ does not exist

(o) $\lim_{n\to\infty} \frac{1+n-2n^2}{1-n+n^2} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n}-2}{\frac{1}{n^2}-\frac{1}{n}+1} = \lim_{n\to\infty} \frac{0+0-2}{0-0+1} = -2$

(p) $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

(q) $\lim_{n\to\infty} \frac{1}{n^5} = 0$

(r) $\lim_{n\to\infty} \frac{1-n^3}{1+2n^3} = \lim_{n\to\infty} \frac{\frac{1}{n^3}-1}{\frac{1}{n^3}+2} = \lim_{n\to\infty} \frac{0-1}{0+2} = -\frac{1}{2}$

(s) $\lim_{n\to\infty} \left(\frac{2}{3}\right)^n = 0$

(t) $\lim_{n\to\infty} \left(\frac{4}{3}\right)^n$ does not exist

4. $\lim_{n\to\infty} t_n = 0.333\,333\,333 \cdots = \frac{1}{3}$

5. $t_n = \frac{2^n}{n^2}$, $t_1 = 2$, $t_2 = 1$, $t_3 = \frac{8}{9}$, $t_4 = 1$, $t_5 = \frac{32}{25}$. $t_6 = \frac{16}{9}$. $t_7 = \frac{128}{49}$, $t_8 = 4$, $t_9 = \frac{512}{81}$, $t_{10} = \frac{1024}{100}$, $t_{20} \doteq 2621.4$, $t_{50} \doteq 4.5\times10^{11}$, $t_{100} \doteq 1.28\times10^{26}$.

$\lim_{n\to\infty} \frac{2^n}{n^2}$ does not exist.

6. $t_n = \sqrt[n]{n}$, $t_1 = 1$, $t_2 = 1.414\,214$, $t_3 = 1.442\,250$, $t_4 = 1.414\,214$, $t_5 = 1.379\,730$, $t_6 = 1.348\,006$, $t_7 = 1.320\,469$, $t_8 = 1.296\,840$, $t_9 = 1.276\,518$, $t_{10} = 1.258\,925$, $t_{50} = 1.081\,383$, $t_{100} = 1.047\,129$, $t_{500} = 1.012\,507$, $t_{1000} = 1.006\,932$, $t_{10\,000} = 1.000\,921$.

$\lim_{n\to\infty} \sqrt[n]{n} = 1$.

7. Let f_n be the number of rabbit pairs in the n^{th} month. We know that $f_1 = 1$, and since the rabbits don't begin to reproduce until the second month, $f_2 = 1$. In the n^{th} month, every pair of rabbits that is at least two months of age (i.e. existed in the $n-2$ generation $\Rightarrow f_{n-2}$ pairs) will add a pair of offspring to the f_{n-1} pairs of rabbits that already exist, so $f_n = f_{n-1} + f_{n-2}$. Thus $\{f_n\}$ is the Fibonacci sequence.

8. $\left\{\sqrt{2},\ \sqrt{2\sqrt{2}},\ \sqrt{2\sqrt{2\sqrt{2}}},\ \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}},\ \ldots\right\}$ can be written as $\left\{2^{\frac{1}{2}},\ \left(2^{\frac{3}{2}}\right)^{\frac{1}{2}},\ \left(2\left(2^{\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}},\right.$

$\left.\left(2\left(2\left(2^{\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}},\ \ldots\right\} = \left\{2^{\frac{1}{2}},\ 2^{\frac{3}{4}},\ 2^{\frac{7}{8}},\ 2^{\frac{15}{16}},\ \ldots\right\} \Rightarrow\ t_n = 2^{\frac{2^n-1}{2^n}}.$

So $\lim\limits_{n\to\infty} t_n = \lim\limits_{n\to\infty} 2^{\frac{2^n-1}{2^n}} = \lim\limits_{n\to\infty} 2^{1-\frac{1}{2^n}} = \lim\limits_{n\to\infty} (2)\left(2^{-\frac{1}{2^n}}\right) = 2(2^0) = 2$

9. (a) $t_1 = 1,\ t_n = \dfrac{1}{2t_{n-1}+1}\ (n \geq 2),\quad t_2 = \frac{1}{3},\ t_3 = \frac{3}{5},\ t_4 = \frac{5}{11},\ t_5 = \frac{11}{21},\ t_6 = \frac{21}{43}$

Guess $\lim\limits_{n\to\infty} t_n = \frac{1}{2}$

(b) $\lim\limits_{n\to\infty} t_n = L \Rightarrow \lim\limits_{n\to\infty} t_{n-1} = L.\qquad \lim\limits_{n\to\infty} t_n = \lim\limits_{n\to\infty} \dfrac{1}{2t_{n-1}+1} \Rightarrow L = \dfrac{1}{2L+1}$

$\Rightarrow 2L^2 + L = 1 \Rightarrow (2L-1)(L+1) = 0 \Rightarrow 2L-1 = 0,\ L+1 = 0 \Rightarrow L = \frac{1}{2},\ L = -1$

L must be positive, so $L = \frac{1}{2}$.

Exercise 1.7

1. (a) $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots \quad a=1,\ r=\frac{1}{3}\ ;\ \ S=\frac{1}{1-\frac{1}{3}}=\frac{3}{2}$

(b) $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\ldots \quad a=1,\ r=-\frac{2}{3}\ ;\ \ S=\frac{1}{1+\frac{2}{3}}=\frac{3}{5}$

(c) $\frac{1}{4}-\frac{5}{16}+\frac{25}{64}-\frac{125}{256}+\ldots \quad a=\frac{1}{4},\ r=-\frac{5}{4}$; Since $|r|\geq 1$, series diverges

(d) $3+\frac{3}{5}+\frac{3}{25}+\frac{3}{125}+\ldots \quad a=3,\ r=\frac{1}{5}\ ;\ \ S=\frac{3}{1-\frac{1}{5}}=\frac{15}{4}$

(e) $1-2+4-8+\ldots \quad a=1,\ r=-2$; Since $|r|\geq 1$, series diverges

(f) $60+40+\frac{80}{3}+\frac{160}{9}+\ldots \quad a=60,\ r=\frac{2}{3}\ ;\ \ S=\frac{60}{1-\frac{2}{3}}=180$

(g) $0.1+0.05+0.025+0.0125+\ldots \quad a=0.1,\ r=\frac{1}{2}\ ;\ \ S=\frac{\frac{1}{10}}{1-\frac{1}{2}}=\frac{1}{5}$

(h) $-3+3-3+3-3+\ldots \quad a=3,\ r=-1$; Since $|r|\geq 1$, series diverges

2. (a) $\sum_{n=1}^{\infty} 2(\frac{3}{4})^{n-1},\quad a=2,\ r=\frac{3}{4}\ ;\ \ S=\frac{2}{1-\frac{3}{4}}=8$

(b) $\sum_{n=1}^{\infty}(-\frac{2}{5})^n=\sum_{n=1}^{\infty}-\frac{2}{5}(-\frac{2}{5})^{n-1},\quad a=-\frac{2}{5},\ r=-\frac{2}{5}\ ;\ \ S=\frac{-\frac{2}{5}}{1+\frac{2}{5}}=-\frac{2}{7}$

3. (a) $0.\bar{1}=\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\frac{1}{10\,000}+\ldots=\frac{\frac{1}{10}}{1-\frac{1}{10}}=\frac{1}{9}$

(b) $0.\overline{25}=\frac{25}{100}+\frac{25}{10\,000}+\frac{25}{1\,000\,000}+\ldots=\frac{\frac{25}{100}}{1-\frac{1}{100}}=\frac{25}{99}$

(c) $0.\overline{41}=\frac{41}{100}+\frac{41}{10\,000}+\frac{41}{1\,000\,000}+\ldots=\frac{\frac{41}{100}}{1-\frac{1}{100}}=\frac{41}{99}$

(d) $0.\overline{157}=\frac{157}{1000}+\frac{157}{1\,000\,000}+\frac{157}{1\,000\,000\,000}+\ldots=\frac{\frac{157}{1000}}{1-\frac{1}{1000}}=\frac{157}{999}$

(e) $1.1\overline{23}=1.1+\frac{23}{1000}+\frac{23}{100\,000}+\frac{23}{10\,000\,000}+\ldots=1.1+\frac{\frac{23}{1000}}{1-\frac{1}{100}}=\frac{11}{10}+\frac{23}{990}=\frac{556}{496}$

(f) $2.3\overline{456}=2.3+\frac{456}{10\,000}+\frac{456}{10\,000\,000}+\frac{456}{10^{10}}+\ldots=2.3+\frac{\frac{456}{10\,000}}{1-\frac{1}{1000}}=\frac{23}{10}+\frac{466}{9990}=\frac{7811}{3330}$

(g) $0.429\overline{113}=0.429+\frac{113}{1\,000\,000}+\frac{113}{10^9}+\frac{113}{10^{12}}+\ldots=0.429+\frac{\frac{113}{1\,000\,000}}{1-\frac{1}{1000}}=\frac{429}{1000}+\frac{113}{999\,000}=\frac{107\,171}{249\,760}$

(h) $6.814\overline{72} = 6.814 + \frac{72}{100\,000} + \frac{72}{10\,000\,000} + \frac{72}{10^9} + \ldots = 6.814 + \frac{\frac{72}{100\,000}}{1-\frac{1}{100}} = \frac{6814}{1000} + \frac{72}{99\,000} = \frac{37\,481}{5500}$

4. (a) $1 + x + x^2 + x^3 + \ldots$; $a = 1$, $r = x$; Converges if $|x| < 1$; $S = \frac{1}{1-x}$

(b) $1 + \frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \ldots$; $a = 1$, $r = \frac{x}{3}$; Converges if $\frac{|x|}{3} < 1 \Rightarrow |x| < 3$;

$S = \frac{1}{1-\frac{x}{3}} = \frac{3}{3-x}$

(c) $1 + \frac{1}{x} + \left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3 + \ldots$; $a = 1$, $r = \frac{1}{x}$; Converges if $\frac{1}{|x|} < 1 \Rightarrow |x| > 1$;

$S = \frac{1}{1-\frac{1}{x}} = \frac{x}{x-1}$

(d) $1 + (x-4) + (x-4)^2 + (x-4)^3 + \ldots$; $a = 1$, $r = (x-4)$; Converges if $|x-4| < 1$

$\Rightarrow 3 < x < 5$; $S = \frac{1}{1-(x-4)} = \frac{1}{5-x}$

(e) $\sum_{n=1}^{\infty} 2^n x^n = \sum_{n=1}^{\infty} 2x(2x)^{n-1}$; $a = 2x$, $r = 2x$; Converges if $|2x| < 1 \Rightarrow |x| < \frac{1}{2}$;

$S = \frac{2x}{1-2x}$

5. $1 - \frac{1}{64} + \frac{1}{729} - \frac{1}{4096} + \ldots + \frac{(-1)^{n-1}}{n^6} + \ldots$; $S_1 = 1$, $S_2 = 0.984\,375$, $S_3 = 0.985\,747$, $S_4 = 0.985\,503$, $S_5 = 0.985\,567$, $S_6 = 0.985\,545$, $S_7 = 0.985\,554$, $S_8 = 0.985\,550$;
Series appears to converge to 0.985 55.

6. $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots + \frac{1}{n(n+1)} + \ldots$; (a) $S_1 = 0.5$, $S_2 = 0.6667$, $S_3 = 0.75$, $S_4 = 0.8$, $S_5 = 0.8333$, $S_6 = 0.8571$, $S_7 = 0.875$, $S_8 = 0.8889$, $S_9 = 0.9$, $S_{10} = 0.9091$, $S_{11} = 0.9167$, $S_{12} = 0.9231$, $S_{13} = 0.9286$, $S_{14} = 0.9333$, $S_{15} = 0.9375$

(b) $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$;

$S_n = \frac{1}{1} - \frac{1}{1+1} + \frac{1}{2} - \frac{1}{2+1} + \frac{1}{3} - \frac{1}{3+1} + \ldots + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1}$

$= 1 - \frac{1}{n+1}$

(c) The sum of the series is the limit of the n^{th} partial sum S_n ;

$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = 1$

(d) $1-\left(1-\frac{1}{n+1}\right)<0.001 \Rightarrow \frac{1}{n+1}<0.001 \Rightarrow 1000<n+1 \Rightarrow n>999$. So at least 1000 terms would be required.

7. $\angle A = \theta$, $AC=1$; Find total length of all perpendiculars $CD+DE+EF+FG+\ldots$ in terms of θ.

$CD = \sin\theta$, $DE = CD\sin\theta = \sin^2\theta$, $EF = DE\sin\theta = \sin^3\theta$; So the total length $= \sin\theta + \sin^2\theta + \sin^3\theta + \ldots$; This is a geometric series with $a=\sin\theta$, $r=\sin\theta$. Since this is a right angled triangle, $\sin\theta<1$. So the sum is $S = \frac{\sin\theta}{1-\sin\theta}$.

Exercise 1.8 Review Exercise

1. From the graph (a) $\lim_{x \to -1} f(x) = 1$ (b) $\lim_{x \to 1^-} f(x) = 0$

(c) $\lim_{x \to 1^+} f(x) = -1$ (d) $\lim_{x \to 1} f(x)$ does not exist (e) $\lim_{x \to -3^+} f(x) = 3$

(f) $\lim_{x \to 4^-} f(x) = 2$ (g) $\lim_{x \to 4^+} f(x) = 2$ (h) $\lim_{x \to 4} f(x) = 2$

2. (a) Discontinuous at 1 (b) Continuous at 4 (c) Discontinuous at 7

3. (a) $\lim_{x \to 2} (3x^3 + 7x - 16) = 3(2)^2 + 7(2) - 16 = 22$

(b) $\lim_{x \to -1} \frac{2x+3}{3x+2} = \frac{2(-1)+3}{3(-1)+2} = -1$ (c) $\lim_{x \to 2} \frac{x^2-2x-8}{x^2-7x+12} = \frac{2^2-2(2)-8}{2^2-7(2)+12} = -4$

(d) $\lim_{x \to 4} \frac{x^2-2x-8}{x^2-7x+12} = \lim_{x \to 4} \frac{(x-4)(x+2)}{(x-4)(x-3)} = \lim_{x \to 4} \frac{x+2}{x-3} = 6$

(e) $\lim_{x \to 5} \sqrt{\frac{x^2-25}{x-5}} = \lim_{x \to 5} \sqrt{\frac{(x+5)(x-5)}{x-5}} = \lim_{x \to 5} \sqrt{x+5} = \sqrt{10}$

(f) $\lim_{x \to 4} \frac{x-4}{x^3-64} = \lim_{x \to 4} \frac{x-4}{(x-4)(x^2+4x+16)} = \lim_{x \to 4} \frac{1}{x^2+4x+16} = \frac{1}{4^2+4(4)+16} = \frac{1}{48}$

(g) $\lim_{t \to 0} \frac{\sqrt{2+t}-\sqrt{2}}{t} = \lim_{t \to 0} \frac{\sqrt{2+t}-\sqrt{2}}{t} \times \frac{\sqrt{2+t}+\sqrt{2}}{\sqrt{2+t}+\sqrt{2}} = \lim_{t \to 0} \frac{2+t-2}{t\left(\sqrt{2+t}+\sqrt{2}\right)}$

$= \lim_{t \to 0} \frac{t}{t\left(\sqrt{2+t}+\sqrt{2}\right)} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

(h) $\lim_{h \to 0} \frac{(-3+h)^2-9}{h} = \lim_{h \to 0} \frac{h^2-6h+9-9}{h} = \lim_{h \to 0} \frac{h^2-6h}{h} = \lim_{h \to 0} (h-6) = -6$

4. (a) $\lim_{x \to -1} \frac{x-6}{(x+1)^3}$ does not exist

(b) $\lim_{x \to 1} \frac{x^2-1}{x^2-7x+6} = \lim_{x \to 1} \frac{(x+1)(x-1)}{(x-6)(x-1)} = \lim_{x \to 1} \frac{x+1}{x-6} = -\frac{2}{5}$

(c) $\lim_{h \to 0} \frac{\frac{4}{2+h}-2}{h} = \lim_{h \to 0} \frac{4-4-2h}{h(2+h)} = \lim_{h \to 0} \frac{-2}{2+h} = -1$

(d) $\lim_{y \to 2} \frac{y^4-16}{y^4+2y^3-y^2-2y} = \frac{2^4-16}{2^4+2(2)^3-2^2-2(2)} = 0$

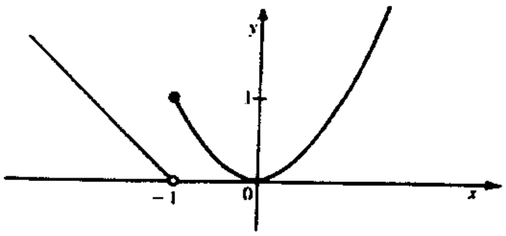
(e) $\lim_{t \to -2^+} \sqrt[4]{8+t^3} = \sqrt[4]{8+(-2)^3} = 0$ (f) $\lim_{x \to 1^+} \frac{|x-1|}{x-1} = \lim_{x \to 1^+} \frac{x-1}{x-1} = \lim_{x \to 1^+} 1 = 1$

(g) $\lim_{x \to 1^-} \frac{|x-1|}{x-1} = \lim_{x \to 1^-} \frac{-(x-1)}{x-1} = -1$ (h) $\lim_{x \to 1} \frac{|x-1|}{x-1}$ does not exist

5. (a) (i) Since $f(x) = -1-x$ for $x < -1$, $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (-1-x) = -1+1 = 0$

(ii) $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x^2 = (-1)^2 = 1$ (iii) $\lim_{x \to -1} f(x)$ does not exist

(b)



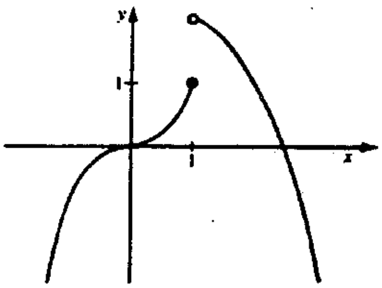
6. (a) (i) Since $g(x) = x^3$ for $x < 0$, $\lim_{x \to 0^-} g(x) = 0^3 = 0$

(ii) Since $g(x) = x^2$ for $0 \le x \le 1$, $\lim_{x \to 0^+} g(x) = 0^2 = 0$ (iii) $\lim_{x \to 0} g(x) = 0$

(iv) Since $g(x) = x^2$ for $0 \le x \le 1$, $\lim_{x \to 1^-} g(x) = 1$

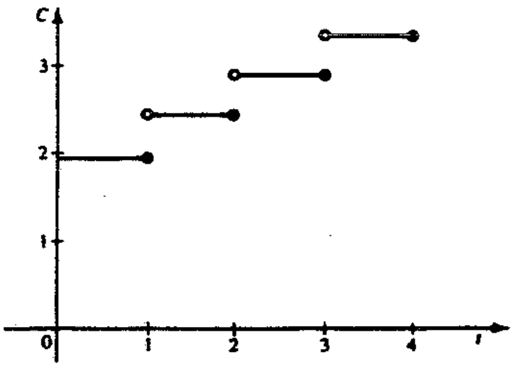
(v) Since $g(x) = 1+2x-x^2$ for $x > 1$, $\lim_{x \to 1^+} g(x) = 2$ (vi) $\lim_{x \to 1} g(x)$ does not exist

(b)



(c) g is discontinuous at 1

7. The function has a discontinuity at the start of every minute, that is at $t = 1, 2, 3, 4, 5$.



8. P(1, −2) lies on $y = x^3 - 3x$

(a) $Q(x, x^3 - 3x)$; slope of PQ, $m = \dfrac{x^3 - 3x + 2}{x - 1}$

(i) $x = 2$, $m = \dfrac{2^3 - 3(2) + 2}{2 - 1} = 4$ (ii) $x = 1.5$, $m = \dfrac{(1.5)^2 - 3(1.5) + 2}{1.5 - 1} = 1.75$

(iii) $x = 1.1$, $m = \dfrac{(1.1)^3 - 2(1.1) + 2}{1.1 - 1} = 0.31$

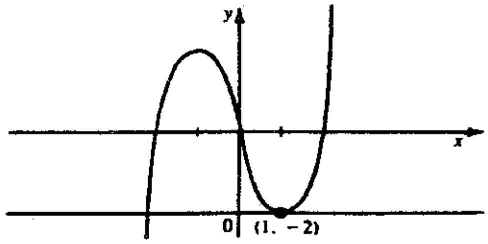
(iv) $x = 1.01$, $m = \dfrac{(1.01)^2 - 3(1.01) + 2}{1.01 - 1} = 0.0301$

(b) Slope of tangent at P, $m = \lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h} = \lim\limits_{h \to 0} \dfrac{(1+h)^3 - 3(1+h) + 2 - 0}{h}$

$= \lim\limits_{h \to 0} \dfrac{1 + 3h + 3h^2 + h^3 - 3 - 3h + 2}{h} = \lim\limits_{h \to 0} (3h + h^2) = 0$

(c) $y + 2 = 0 \Rightarrow y = -2$

(d)



9. $y = x^4$, $(-1, 1)$; $m = \lim\limits_{h \to 0} \dfrac{f(-1+h) - f(-1)}{h} = \lim\limits_{h \to 0} \dfrac{(-1+h)^4 - 1}{h}$

$= \lim\limits_{h \to 0} \dfrac{h^4 - 4h^3 + 6h^2 - 4h + 1 - 1}{h} = \lim\limits_{h \to 0} (h^3 - 4h^2 + 6h - 4) = -4.$

So $(y - 1) = -4(x + 1) \Rightarrow 4x + y + 3 = 0$ is the equation of the tangent line.

10. $h = s(t) = 200 - 4.9t^2$; (a) $\bar{v} = \dfrac{200 - 4.9t^2 - [200 - 4.9]}{t - 1} = \dfrac{4.9 - 4.9t^2}{t - 1}$

(i) $1 \le t \le 2$, $\bar{v} = \dfrac{4.9 - 4.9(2)^2}{2 - 1} = -14.7$ m/s

(ii) $1 \le t \le 1.1$, $\bar{v} = \dfrac{4.9 - 4.9(1.1)^2}{1.1 - 1} = -10.3$ m/s

(b) $v(1) = \lim\limits_{h \to 0} \dfrac{s(1+h) - s(1)}{h} = \lim\limits_{h \to 0} \dfrac{200 - 4.9(1+h)^2 - [200 - 4.9]}{h}$

$= \lim\limits_{h \to 0} \dfrac{200 - 4.9h^2 - 9.8h - 4.9 - 195.1}{h} = \lim\limits_{h \to 0} (-4.9h - 9.8) = -9.8$ m/s

11. $r = 10$ cm, $S = 4\pi r^2$, $\lim\limits_{\Delta r \to 0} \dfrac{\Delta S}{\Delta r} = \lim\limits_{r \to 10} \dfrac{S(r) - S(10)}{r - 10} = \lim\limits_{r \to 10} \dfrac{4\pi r^2 - 4\pi(100)}{r - 10}$

$= \lim\limits_{r \to 10} \dfrac{4\pi(r + 10)(r - 10)}{r - 10} = \lim\limits_{r \to 10} 4\pi(r + 10) = 80\pi$ cm²/cm

12. (a) $\lim_{n\to\infty}\left(2-\frac{1}{n}+\frac{3}{n^2}\right) = 2-0+0 = 2$

(b) $\lim_{n\to\infty}\frac{1+2n}{1-3n} = \lim_{n\to\infty}\frac{\frac{1}{n}+2}{\frac{1}{n}-3} = \frac{0+2}{0-3} = -\frac{2}{3}$

(c) $\lim_{n\to\infty}(1.1)^n$ does not exist

(d) $\lim_{n\to\infty}\frac{3^n}{5^n} = \lim_{n\to\infty}\left(\frac{3}{5}\right)^n = 0$

13. (a) $6-1+\frac{1}{6}-\frac{1}{36}+\dots$; $a=6$, $r=-\frac{1}{6} \Rightarrow S = \frac{6}{1+\frac{1}{6}} = \frac{36}{7}$

(b) $\frac{1}{9}+\frac{1}{3}+1+3+\dots$; $a=\frac{1}{6}$. $r=3$, Since $|r|\geq 1$, the series diverges.

14. $1.2\overline{45} = 1.2+\frac{45}{1000}+\frac{45}{100\,000}+\frac{45}{10\,000\,000}+\dots = 1.2+\frac{\frac{45}{1000}}{1-\frac{1}{100}} = \frac{12}{10}+\frac{45}{990} = \frac{137}{110}$

15. $\sum_{n=1}^{\infty}(x+1)^n = \sum_{n=1}^{\infty}(x+1)(x+1)^{n-1}$, so the sum converges when $|x+1|<1$
$\Rightarrow -2<x<0$. $a=x+1$, $r=x+1 \Rightarrow S = \frac{x+1}{1-(x+1)} = -\frac{x+1}{x} = -1-\frac{1}{x}$

16. $t_1 = \sqrt{3} = 3^{\frac{1}{2}}$. $t_{n+1} = \sqrt{3t_n}$, $(n\geq 1)$. Find $\lim_{n\to\infty} t_n$.

Note that $t_2 = \sqrt{3\sqrt{3}} = 3^{\frac{3}{4}}$. $t_3 = \sqrt{3(3^{\frac{3}{4}})} = 3^{\frac{7}{8}}, \dots, t_n = 3^{\frac{2^n-1}{2^n}}$.

So $\lim_{n\to\infty} t_n = \lim_{n\to\infty} 3^{\frac{2^n-1}{2^n}} = \lim_{n\to\infty} 3^{1-\frac{1}{2^n}} = 3.$

Exercise 1.9 Chapter 1 Test

1. (a) $\lim_{x \to 2} \sqrt{\frac{x^2+5}{x-1}} = \sqrt{\frac{2^2+5}{2-1}} = 3$

(b) $\lim_{x \to -1} \frac{x+1}{x^2-4x-5} = \lim_{x \to -1} \frac{x+1}{(x+1)(x-5)} = \lim_{x \to -1} \frac{1}{x-5} = -\frac{1}{6}$

(c) $\lim_{x \to 1} \frac{\frac{1}{\sqrt{x}}-1}{x-1} = \lim_{x \to 1} \frac{\frac{1}{\sqrt{x}}-1}{x-1} \times \frac{\frac{1}{\sqrt{x}}+1}{\frac{1}{\sqrt{x}}+1}$

$= \lim_{x \to 1} \frac{\frac{1}{x}-1}{\sqrt{x}+x-\frac{1}{\sqrt{x}}-1} = \lim_{x \to 1} \frac{\frac{1}{x}-1}{-\sqrt{x}\left(\frac{1}{x}-1\right) - x\left(\frac{1}{x}-1\right)} = \lim_{x \to 1} \frac{1}{-\sqrt{x}-x} = -\frac{1}{2}$

2. P(2, −1), Q(3, −4), $y = -x^2+2x-1$

(a) Slope PQ $= \frac{-4+1}{3-2} = -3$

(b) Slope of tangent at P

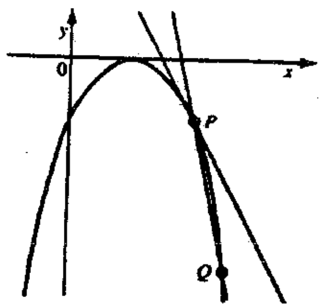
$m = \lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h \to 0} \frac{-(2+h)^2+2(2+h)-1-[-(2)^2+2(2)-1]}{h}$

$= \lim_{h \to 0} \frac{-h^2-4h-4+4+2h-1+4-4+1}{h} = \lim_{h \to 0} \frac{-h^2-2h}{h}$

$= \lim_{h \to 0} (-h-2) = -2$

(c) $y+1 = -2(x-2) \Rightarrow y = -2x+3$ or $2x+y-3 = 0$

(d)

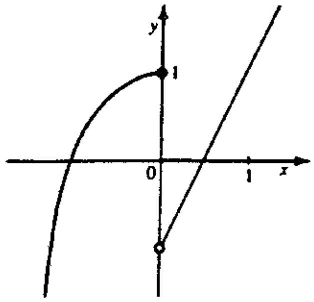


3. (a) (i) Since $f(x) = 1-x^2$ for $x \le 0$, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (1-x^2) = 1-0^2 = 1$

(ii) Since $f(x) = 2x-1$ for $x > 0$, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} [2(0)-1] = -1$

(iii) $\lim_{x \to 0} f(x)$ does not exist

(b)



(c) f is discontinuous at 0

4. $s = 5t^2 - 6t + 14$

(a) $2 \leq t \leq 3$, $\bar{v} = \frac{s(3) - s(2)}{3-2} = 5(3)^2 - 6(3) + 14 - [5(2)^2 - 6(2) + 14] = 19$ m/s

(b) $v(2) = \lim_{h \to 0} \frac{s(2+h) - s(2)}{h} = \lim_{h \to 0} \frac{5(2+h)^2 - 6(2+h) + 14 - [5(2)^2 - 6(2) + 14]}{h}$

$= \lim_{h \to 0} \frac{20 + 20h + 5h^2 - 12 - 6h + 14 - 22}{h} = \lim_{h \to 0} \frac{14h + 5h^2}{h}$

$= \lim_{h \to 0} (14 + 5h) = 14$ m/s

5. $\lim_{n \to \infty} \left(\frac{1}{8^n} + \frac{6n-2}{2n-3}\right) = \lim_{n \to \infty} \left(\frac{1}{8^n}\right) + \lim_{n \to \infty} \frac{6n-2}{2n-3} = 0 + \lim_{n \to \infty} \frac{6 - \frac{2}{n}}{2 - \frac{3}{n}} = \frac{6-0}{2-0} = 3$

6. $12 - 9 + \frac{27}{4} - \frac{81}{16} + \ldots$; $a = 12$, $r = -\frac{3}{4} \Rightarrow S = \frac{12}{1 + \frac{3}{4}} = \frac{48}{7}$

Review and Preview to Chapter 2

EXERCISE 1

1. (a) $f(x)=1-18x,\ x\in R$

(b) $g(x)=x^4-x^2+15x,\ x\in R$

(c) $h(x)=\sqrt{x-5},\ \{x \mid x-5\geq 0\}=\{x \mid x\geq 5\}$

(d) $F(x)=\sqrt[4]{-x},\ \{x \mid -x\geq 0\}=\{x \mid x\leq 0\}$

(e) $G(x)=\sqrt{1-x^2},\ \{x \mid 1-x^2\geq 0\}=\{x \mid x^2\leq 1\}=\{x \mid |x|\leq 1\}=\{x \mid -1\leq x\leq 1\}$

(f) $H(x)=\sqrt{x^2-2},\ \{x \mid x^2-2\geq 0\}=\{x \mid x^2\geq 2\}=\{x \mid |x|\geq \sqrt{2}\}$

$=\{x \mid x\geq\sqrt{2} \text{ or } x\leq -\sqrt{2}\}$

(g) $y=\dfrac{3+x}{3-x},\ \{x \mid 3-x\neq 0\}=\{x \mid x\neq 3\}$

(h) $y=\dfrac{x^2}{x^2+4x-5},\ \{x \mid x^2+4x-5\neq 0\}=\{x \mid (x+5)(x-1)\neq 0\}$

$=\{x \mid x\neq -5,\ x\neq 1\}$

(i) $y=\dfrac{1}{\sqrt{t^2+5}},\ \{t \mid t^2+5>0\}\Rightarrow t\in R$

(j) $y=\dfrac{t}{\sqrt{t^2-5t+6}},\ \{t \mid t^2-5t+6>0\}=\{t \mid (t-2)(t-3)>0\}=\{t \mid t<2 \text{ or } t>3\}$

(k) $f(x)=\sqrt{x}+\sqrt{4-x},\ \{x \mid x\geq 0 \text{ and } 4-x\geq 0\}=\{x \mid 0\leq x\leq 4\}$

(l) $f(x)=\sqrt{2-\sqrt{4-x}},\ \{x \mid 4-x\geq 0 \text{ and } \sqrt{4-x}\leq 2\}=\{x \mid x\leq 4 \text{ and } 4-x\leq 4\}$

$=\{x \mid 0\leq x\leq 4\}$

EXERCISE 2

1. (a) $f(x)=2x-1,\ g(x)=4-3x,$

$(f\circ g)(x)=f(g(x))=f(4-3x)=2(4-3x)-1=7-6x$

$(g\circ f)(x)=g(f(x))=g(2x-1)=4-3(2x-1)=7-6x$

$(f\circ f)(x)=f(f(x))=f(2x-1)=2(2x-1)-1=4x-3$

$(g\circ g)(x)=g(g(x))=g(4-3x)=4-3(4-3x)=9x-8$

(b) $f(x)=x^2,\ g(x)=x+1$

$(f\circ g)(x)=f(x+1)=(x+1)^2=x^2+2x+1$

$(g\circ f)(x)=g(x^2)=x^2+1$

$(f\circ f)(x)=f(x^2)=(x^2)^2=x^4$

$(g\circ g)(x)=g(x+1)+1=x+2$